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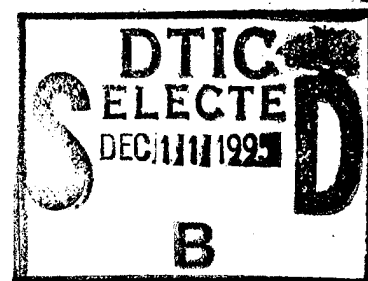
Three-Dimensional Effective Property and Strength Prediction of Thick Laminated Composite Media

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13. ABSTRACT (Maximum 200 words) An analytical model appropriate for the analysis and design of thick laminated composite media is presented. The model is based on the theoretical work of Chou, Carleone, and Hsu (1972) and predicts the effective three-dimensional thermo-mechanical properties and ultimate strength of thick laminated composite media under applied mechanical and thermal loadings. Various types of lamina constitutive models and lamina failure criteria (including progressive ply failure) have been incorporated in the analysis. A major result of this work is the codification of a three-dimensional laminated media model into a user-friendly computer program environment (LAM3D) for implementation as an engineering design tool for thick laminated composite structures. LAM3D predictions are validated with comparisons between results in the published literature and available experimental data. Several illustrative examples are presented to demonstrate the utility of the code for predicting the effective properties and ultimate strength of laminated composite media. This work represents an important step toward the development of a computationally efficient engineering tool which will significantly enhance the design and analysis capability for the development of a wide range thick composite structures currently being investigated for military and commercial applications.					
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THREE-DIMENSIONAL EFFECTIVE PROPERTY AND STRENGTH PREDICTION OF THICK LAMINATED COMPOSITE MEDIA

1 INTRODUCTION

1.1 Objective and Motivation

The objective of this work was to develop an engineering design tool capable of determining the effective three-dimensional thermo-mechanical properties and ultimate strength characteristics of laminated composite media. Specifically, a suitable homogeneous representation is sought to replace the actual heterogeneity of the laminated composite material. Homogeneous representations in the form of three-dimensional anisotropic stiffness matrices and the typical thermo-mechanical engineering constants of laminated media are developed. In addition, a laminate failure analysis that identifies specific ply failure modes and provides ultimate laminate strength predictions (defined in terms of average or effective laminate stress) has been incorporated in the predictive capabilities.

Modeling ply-level details is often an unrealistic approach in the design and analysis of multi-layered laminated composite structures because of computational limitations and time constraints. Consequently, homogeneous (or "smeared") material representations for "thick" laminated materials are often highly useful. Such representations, when used in conjunction with traditional structural analysis techniques (e.g., the finite element method), enable accurate prediction of the effective or "average" stress and strain distributions that develop within a laminated composite structure under mechanical and/or thermal loading (Chou, Carleone, and Hsu 1972).

The failure analysis of thick laminated composite structures should not, however, be based solely on average stress and/or strain distributions. In practice, it is a generally accepted fact that any realistic laminate failure assessment must be based on ply-level stress or strain results. A reasonable approximation of local ply-level stress-strain results can be obtained, however, by applying the average stress-strain state as a local set of boundary conditions onto a laminated media boundary value problem. In this manner, ply-level stress-strain details throughout an entire structure can be computed and subsequently employed in a realistic failure assessment methodology.

One such approach, which is sometimes referred to as "smearing-unsmeared," is depicted in Figure 1. A representative sublaminar configuration for the composite structure is first

identified (see Step 1 in Figure 1). In the most general sense, this material could be considered as heterogeneous and anisotropic. A set of equivalent or effective homogeneous properties for this representative sublamine configuration is then computed (see Step 2 in Figure 1). This step is referred to as the “smearing” of the properties. Although this is typically done with an analytical solution technique (e.g., classical laminated plate theory [Whitney 1987]), a numerical approach may also be employed.

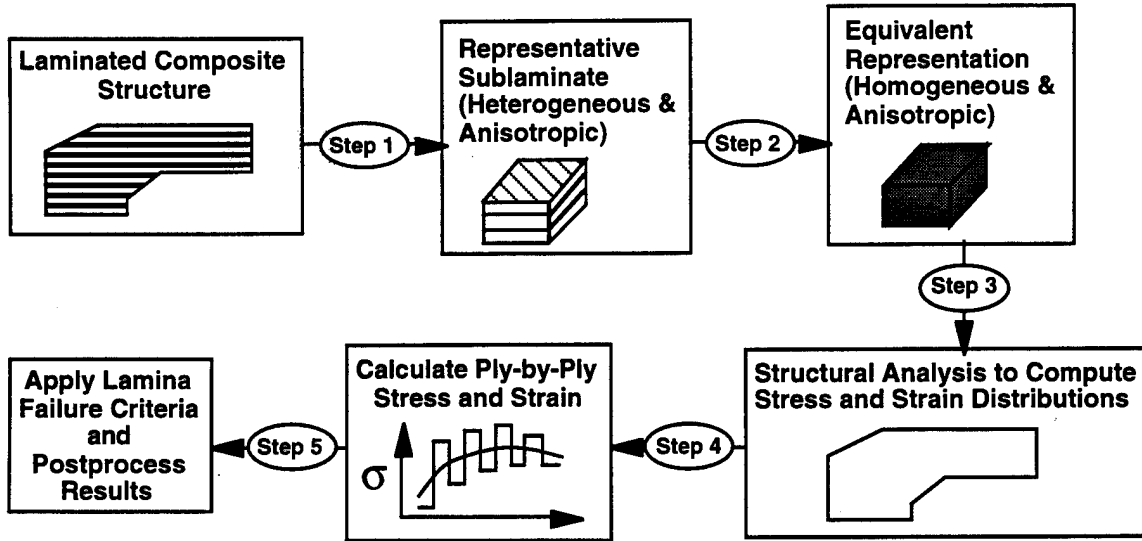


Figure 1. Smearing-unsmeared analysis methodology.

Next, the assumption is made that the effective or homogeneous thermo-mechanical properties of the representative sublamine configuration can be used to replace the actual heterogeneity of the laminated material in the structural analysis. A typical structural analysis is then conducted, employing the effective thermo-mechanical properties as input to obtain the average stress and strain distributions within the structure under a prescribe loading (see Step 3 in Figure 1). It is important to note that the stress and strain distributions here represent the average values (i.e., smooth and continuous), since the laminated material has been replaced with an equivalent set of homogeneous properties. They do not reflect the actual stress and strain distributions (i.e., ply by ply) associated with the heterogeneity of the material. The ply-level stress and strains states are required to accurately assess failure within the laminated composite structure.

An assessment of ply-level stress and strain results can be obtained straightforwardly by applying the average stress-strain state as a local set of boundary conditions onto a laminated media boundary value problem. Again, this can be conducted analytically or numerically (see Step 4 in Figure 1). This step is referred to as the “unsmeared” of the average stress-strain state. Ultimately, the local ply-level stress-strain states throughout the structure can be calculated and employed in an appropriate lamina failure criterion to assess overall mechanical performance (see Step 5 in Figure 1) of the structure.

A major result of this work is the codification of a three-dimensional laminated media model into a user-friendly computer program environment (LAM3D) for implementation as an engineering design and analysis tool for thick laminated composite structures. In the subsequent sections, the theoretical background of the laminated media theory and failure analysis employed in LAM3D are presented. Effective thermo-mechanical property predictions and ultimate laminate strength estimates from LAM3D are validated with results given in the published literature and available experimental results. Several illustrative example problems are presented to summarize typical input and output parameters required by LAM3D and to demonstrate the utility of the code for general laminate analysis purposes.

1.2 Problem Statement

In this work, the analytical model developed by Chou, Carleone, and Hsu (1972) is used to (1) compute the effective (homogeneous) three-dimensional stress-strain constitutive relations and thermo-mechanical properties of a laminated media composed of an arbitrary number of orthotropic layers or plies, (2) compute the three-dimensional ply-level stress and strain states within the laminated media subjected to three-dimensional mechanical loading and/or uniform thermal loading, and (3) generate ultimate laminate strength predictions based on various lamina failure criteria and a progressive ply failure analysis methodology.

Three different lamina constitutive relations are considered: (1) a general orthotropic material assumption that requires nine independent elastic constants, (2) a transversely isotropic material assumption that requires five independent elastic constants, and (3) a simplified form of the transversely isotropic case requiring only three independent elastic constants. The first two material constitutive relations require in-plane and out-of-plane material properties to describe the three-dimensional stiffness matrix for a single lamina. The third material model only requires in-plane properties and is therefore useful in practical applications when three-dimensional material properties may not be available because of limited experimental characterization data.

A total of eight lamina failure criteria have been incorporated into the failure analysis to predict ultimate laminate strength at first ply failure or in the presence of progressive ply failure. Thermal residual ply stresses attributable to uniform or non-uniform thermal laminate loading have also been incorporated in the strength predictions. Ultimate laminate strengths as well as the particular modes of failure in the individual plies are predicted.

1.3 Literature Survey

Much attention has been given to determining the three-dimensional effective or homogeneous properties of laminated media for structural analysis purposes. This literature survey presents an overview of relevant research conducted in this area.

Early efforts focused on obtaining equivalent material stiffness constants for laminated media composed of individual layers that are isotropic (White and Angona 1955; Postma 1955; Rytov 1956; Behrens 1967). Salamon (1968) conducted a similar analysis but computed effective compliance constants for the laminated media. For the most part, the approaches taken in these early works employed dispersion techniques and elastic wave propagation theories. These researchers found the effective stiffness constants of the laminated media to possess transverse isotropy.

Sun, Achenbach, and Hermann (1968) have also investigated the problem of effective property determination of laminated materials. They have taken a continuum theory approach in which the microstructure of an alternately layered medium is modeled. Achenbach (1970) later showed how the results of the continuum theory approach reduce to earlier generated results under appropriate assumptions related to the details of the microstructure.

Christensen (1988) developed a three-dimensional lamina constitutive relationship based on only three independent properties rather than the five typically associated with transversely isotropic material models. He used this simplified lamina constitutive theory to develop a three-dimensional laminate theory that closely resembles the classical two-dimensional laminated plate theory for the in-plane directions. Out-of-plane directional properties for the laminate are identical to those of any individual layer since they are assumed to be independent of in-plane ply orientations. These results led directly to a three-dimensional lamina failure theory that separates fiber and matrix dominated failure modes. The composite lamina failure theory closely resembles the Von Mises criterion often used for isotropic materials.

Enie and Rizzo (1970) developed a convenient set of expressions for determining the effective three-dimensional mechanical properties of balanced laminates. They employed classical laminated plate theory for calculating in-plane properties. Out-of-plane properties were predicted, based on uniform interlaminar shear and interlaminar normal stress distributions and uniform in-plane strains assumptions. Model predictions were found to be in reasonable agreement with finite element results for several different composite laminate configurations.

Pagano (1974a, 1974b) developed a three-dimensional laminate model within the framework of anisotropic heterogeneous elasticity theory to predict the effective response of composite laminates to mechanical and thermal loading. His work, which was not restricted to isotropic layers or the assumption of a repeating sublaminate, considered the representative volume to contain the entire thickness of the laminate. Laminate symmetry was also not required, and consequently, his constitutive "stress-strain" relations included moment and curvature effects.

Chou, Carleone, and Hsu (1972) conducted a control volume approach to yield a closed form solution to the problem of effective homogeneous property determination for a laminated media composed of individual layers. Unlike the works of White and Angona (1955), Postma (1955), Rytov (1956), Behrens (1967) and Salamon (1968), which required the individual layers to be isotropic, Chou, Carleone, and Hsu (1972) permitted general anisotropy of the layers. The analysis is based on the assumptions that all interlaminar stresses are continuous across ply interfaces and that all in-plane strains are continuous through the thickness dimension of the representative volume element (i.e., the repeating sub-laminate configuration).

Sun and Li (1988) developed closed form expressions to predict the equivalent homogeneous anisotropic representation of a laminated composite medium. Their analysis is remarkably similar to the earlier work presented by Chou, Carleone, and Hsu (1972). Reduced expressions for the elastic constants are derived, which greatly simplify the analysis. Effective property predictions for several laminates are generated and found to be in reasonable agreement with the results of Enie and Rizzo (1970).

Roy and Tsai (1992) formulated a boundary value problem approach and used the Airy stress function solution technique to estimate three-dimensional effective properties for laminated composite media. They note that since the works of Pagano (1974a, 1974b); Chou, Carleone, and Hsu (1972); and Sun and Li (1988) assume the distributions of interlaminar shear stress to be constant through the thickness of the laminate, their effective interlaminar shear

stiffnesses are independent of stacking sequence, which is not in complete agreement with the experimental observations of Roy and Kim (1989).

The model developed by Chou, Carleone, and Hsu (1972) was selected for implementation in the present work, based on the combined merits of its accuracy and computational efficiency. Their model predictions have been validated with the results of other investigators. In addition, their model provides a convenient closed form solution for enhanced computational efficiency, which is a strong consideration in future work for the development of an engineering tool for the design and analysis of thick laminated composite structures (e.g., finite element pre- and post-processors).

2. ANALYSIS

2.1. Nomenclature

The laminate nomenclature adopted in this work, including ply stress and ply strain notation, is consistent with that in most composite material primers for two-dimensional laminate analyses (Whitney, Daniel, and Pipes 1982; Whitney 1987). The three-dimensional version of this notation is reviewed in this section.

An orthogonal principal material coordinate system (1, 2, 3) of a single ply, or lamina, is illustrated in Figure 2. The principal material directions include the fiber or longitudinal (1-direction), the transverse (2-direction) and the normal (3-direction). A laminate or representative sublaminar configuration is composed of N individual plies and is defined with respect to an orthogonal (x, y, z) global laminate coordinate system (see Figure 3). The (1,2) plane of each ply is assumed to be co-planar with the x - y laminate plane. The orientation angle of each ply, θ , defines the right-hand rotation of the (1, 2, 3) ply coordinate system about the 3 axis with respect to the fixed (x, y, z) system (see Figure 3). The “stacking sequence” or “lay-up” of the laminate describes the orientation pattern of all plies in the laminate. The geometric mid-plane of the laminate is positioned at the $z=0$ location. The bottom ply is designated as Ply 1 and the top ply is designated as Ply N . A notation is used to define the lay-up in which ply angles are separated by slashes, and brackets are used to enclose all the plies in the laminate. For example, the nomenclature $[45/90/0/-45/-45/0/90/45]$ defines stacking sequence for the eight-ply laminate depicted in Figure 3.

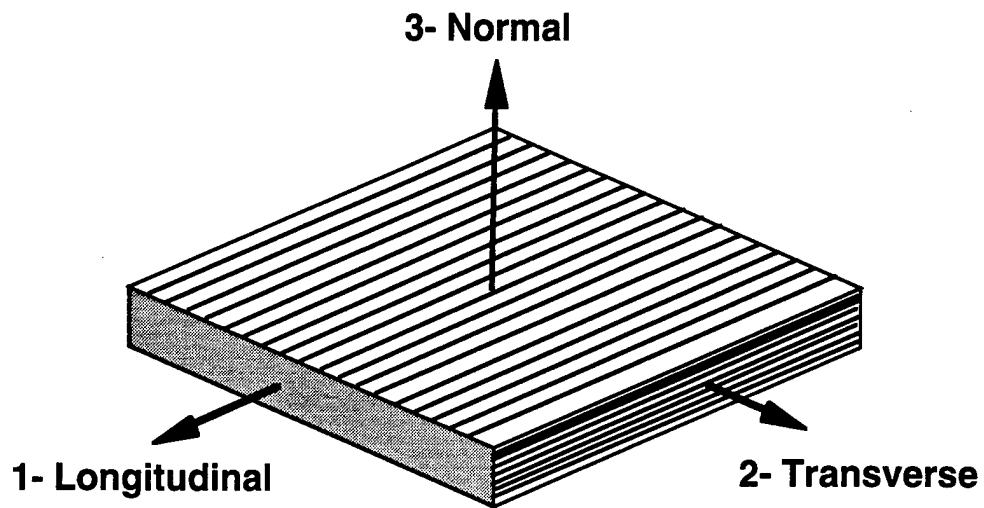


Figure 2. Lamina (principal material) coordinate system.

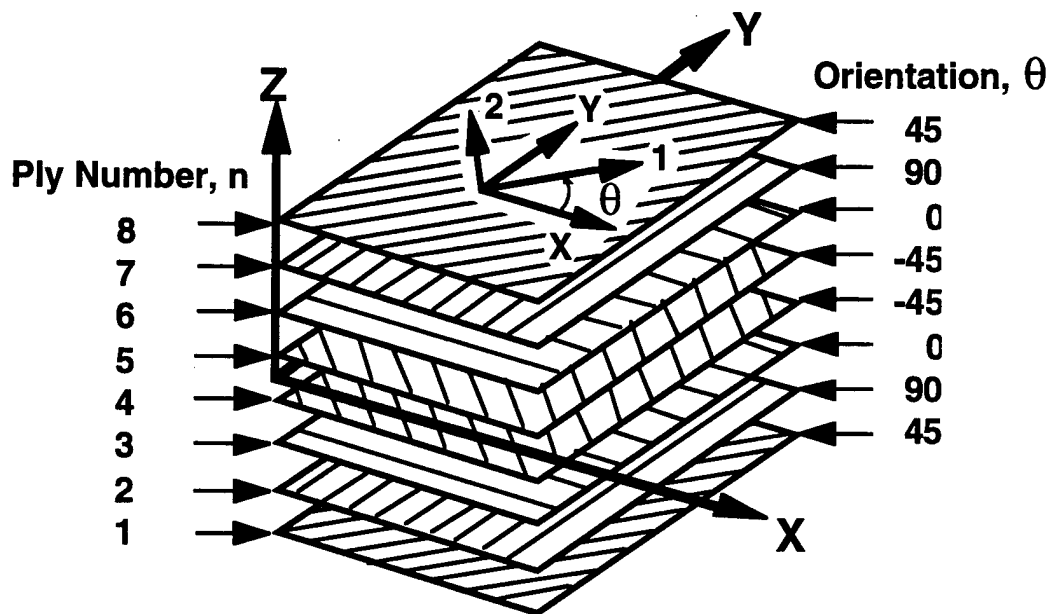


Figure 3. Laminate nomenclature.

When stress and strain are referenced in the principal (1, 2, 3) material coordinate system, they are denoted with an accent and are represented according to the following conventions. For stress, the notation is given by

$$\sigma'_i = [\sigma'_{11} \ \sigma'_{22} \ \sigma'_{33} \ \sigma'_{23} \ \sigma'_{13} \ \sigma'_{12}]^T \quad \text{for} \quad (i = 1, 6) \quad (1)$$

Similarly, the notation for strain is given by

$$\epsilon'_i = [\epsilon'_{11} \ \epsilon'_{22} \ \epsilon'_{33} \ \epsilon'_{23} \ \epsilon'_{13} \ \epsilon'_{12}]^T \quad \text{for} \quad (i = 1, 6) \quad (2)$$

The subscripts 11, 22, and 12 refer to the longitudinal, transverse and shear components, respectively, of both in-plane stress and strain. The subscripts 33, 23, and 13 refer to the interlaminar normal and two interlaminar shear components, respectively, of the out-of-plane stress and strain. This contracted notation is schematically illustrated in Figure 4. for the stresses acting on the faces of an idealized material element of a unidirectional ply. The notation for ply strains would be represented in an analogous manner.

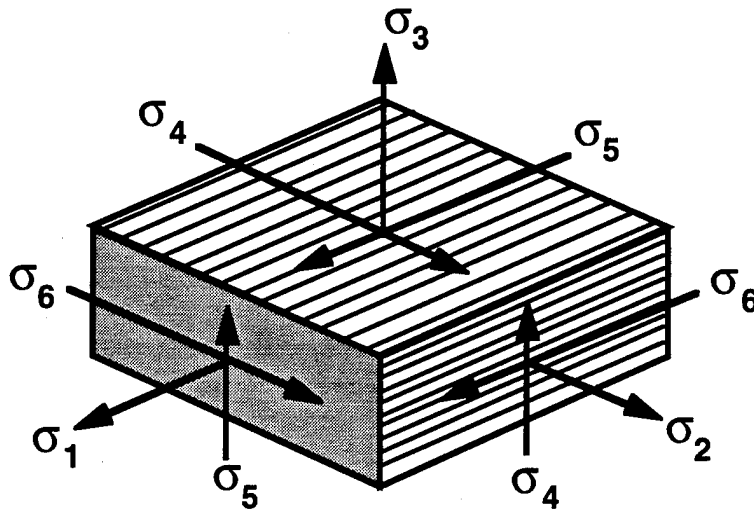


Figure 4. Contracted notation for stress.

$$\bar{\sigma}_i = [\bar{\sigma}_x \ \bar{\sigma}_y \ \bar{\sigma}_z \ \bar{\sigma}_{yz} \ \bar{\sigma}_{xz} \ \bar{\sigma}_{xy}]^T \quad \text{for} \quad (i = 1, 6) \quad (3)$$

and strains by

$$\bar{\epsilon}_i = [\bar{\epsilon}_x \ \bar{\epsilon}_y \ \bar{\epsilon}_z \ \bar{\epsilon}_{yz} \ \bar{\epsilon}_{xz} \ \bar{\epsilon}_{xy}]^T \quad \text{for} \quad (i = 1, 6) \quad (4)$$

The subscripts x, y, and xy represent the in-plane components of stress and strain defined in the global (x, y, z) laminate coordinate system. The subscripts z, yz, and xz represent the out-of-plane components of stress and strain.

2.2 Lamina Constitutive Relations

The three-dimensional Hooke's Law linear-elastic stress-strain constitutive relation for an individual lamina is written in the following contracted form

$$\sigma'_i = C'_{ij} \epsilon'_j \quad \text{for} \quad (i, j = 1, 2, 3, 4, 5, 6) \quad (5)$$

in which C'_{ij} represents the stiffness matrix of the material defined in the principal material coordinate system. For a material that possesses general anisotropy, 21 independent elastic constants are required to characterize the stiffness matrix. Various assumptions about the material symmetry of continuous fiber-reinforced composites can be made to reduce the number of independent constants required to characterize the lamina stiffness matrix. The three types of lamina constitutive models considered in this work, which are simplifications of the general anisotropic case, are summarized in the following sections.

2.2.1 Orthotropic Material Model

An individual lamina of unidirectional composite material possess three mutually orthogonal planes of elastic symmetry (see Figure 2) and is said to be "orthotropic." The orthotropic lamina constitutive model is the most general type of material model considered in this work. The elastic symmetry of orthotropic materials reduces the number of independent elastic constants required to characterize the lamina stiffness matrix from 21 to 9. These elastic constants are defined in terms of the following typical engineering properties: the extensional moduli E_1 , E_2 , and E_3 ; the shear moduli G_{23} , G_{13} , and G_{12} ; and the Poisson's ratios ν_{23} , ν_{13} , and ν_{12} . The orthotropic lamina stiffness matrix is symmetrical (i.e., $C'_{ij} = C'_{ji}$ for $i, j = 1, 2, 3, 4, 5, 6$) and takes the following form (Whitney, 1987):

$$C'_{ij} = \begin{bmatrix} C'_{11} & C'_{12} & C'_{13} & 0 & 0 & 0 \\ C'_{12} & C'_{22} & C'_{23} & 0 & 0 & 0 \\ C'_{13} & C'_{23} & C'_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C'_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C'_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C'_{66} \end{bmatrix} \quad (6)$$

The non-zero stiffness coefficients of the orthotropic lamina stiffness matrix are defined as follow:

$$\begin{aligned} C'_{11} &= (1 - \nu_{23}^2 E_3 / E_2) E_1 / V \\ C'_{12} &= (\nu_{12} + \nu_{13} \nu_{23} E_3 / E_2) E_2 / V \\ C'_{13} &= (\nu_{13} + \nu_{12} \nu_{23}) E_3 / V \\ C'_{22} &= (1 - \nu_{13}^2 E_3 / E_1) E_2 / V \\ C'_{23} &= (\nu_{23} - \nu_{12} \nu_{13} E_2 / E_1) E_3 / V \\ C'_{33} &= (1 - \nu_{12}^2 E_2 / E_1) E_3 / V \\ C'_{44} &= G_{23} \\ C'_{55} &= G_{13} \\ C'_{66} &= G_{12} \end{aligned} \quad (7)$$

in which

$$V = 1 - \nu_{12} (\nu_{12} E_2 / E_1 + 2 \nu_{23} \nu_{13} E_3 / E_1) - \nu_{13}^2 E_3 / E_1 - \nu_{23}^2 E_3 / E_2$$

2.2.2 Transversely Isotropic Material Model

It is generally acceptable to assume that the composite lamina also possess material symmetry in the (2, 3) plane of the principal material coordinate system (see Figure 2). The material is therefore said to be “transversely isotropic” and the following conditions are assumed to hold true:

$$\begin{aligned} E_3 &= E_2 \\ G_{13} &= G_{12} \\ \nu_{13} &= \nu_{12} \end{aligned} \quad (8)$$

In this case, the number of independent elastic constants is reduced from nine (orthotropic) to five (transversely isotropic). The lamina stiffness matrix for the transversely isotropic material model takes the same form as that given in equation (6). Using a subset of the coefficients defined in equation (7), the matrix is written as (Whitney, 1987)

$$C'_{ij} = \begin{bmatrix} C'_{11} & C'_{12} & C'_{12} & 0 & 0 & 0 \\ C'_{12} & C'_{22} & C'_{23} & 0 & 0 & 0 \\ C'_{12} & C'_{23} & C'_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{C'_{22}-C'_{23}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & C'_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & C'_{66} \end{bmatrix} \quad (9)$$

Now the reduced set of engineering constants required to characterize the lamina stiffness matrix includes only E_1 , E_2 , G_{12} , ν_{12} , and ν_{23} .

2.2.3 Simplified Transversely Isotropic Material Model

A further reduction in the number of elastic constants required to characterize the lamina stiffness matrix is also possible (Christensen, 1988). Christensen has suggested that the number of elastic constants may be reduced to only three, using the following two restrictions on the lamina stiffness matrix coefficients

$$\frac{C'_{22}-C'_{23}}{2} = C'_{66} \quad \text{and} \quad C'_{23} = C'_{12} \quad (10)$$

In terms of the engineering properties, these restrictions or conditions are met if the following relationship is found true:

$$G_{12} = \frac{\{1-\nu_{12}\} E_2}{2 \left\{1 - \frac{\nu_{12}^2 E_2}{E_1}\right\}} \quad (11)$$

Christenson (1988) showed, with experimental evidence, that these assumptions are valid for several typical composite material systems. If Equation (11) applies, the lamina stiffness matrix assumes the following form:

$$C'_{ij} = \begin{bmatrix} C'_{11} & C'_{12} & C'_{12} & 0 & 0 & 0 \\ C'_{12} & C'_{22} & C'_{12} & 0 & 0 & 0 \\ C'_{12} & C'_{12} & C'_{22} & 0 & 0 & 0 \\ 0 & 0 & 0 & C'_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & C'_{66} & 0 \\ 0 & 0 & 0 & 0 & 0 & C'_{66} \end{bmatrix} \quad (12)$$

in which only the in-plane engineering properties E_1 , E_2 , and ν_{12} are required to characterize the lamina stiffness matrix. Motivation for this material model is because empirical characterization of the in-plane material properties is significantly more straightforward than that of the out-of-plane properties.

2.3 Transformations Between the Principal and Laminate Coordinate Systems

Transformation of ply stress and strain between the principal (1, 2, 3) and global (x,y,z) coordinate systems is required. The global ply stresses, $\bar{\sigma}_i$, can be expressed explicitly in terms of the principal ply stresses, σ'_i , and the ply orientation angle, θ , (see Figure 3).

Mathematically, this transformation is accomplished with the following second order tensor transformation:

$$\bar{\sigma}_i = [T(\theta)]_{ij}^{\sigma} \sigma'_j \quad (13)$$

in which the stress transformation matrix is given by

$$[T(\theta)]_{ij}^{\sigma} = \begin{bmatrix} m^2 & n^2 & 0 & 0 & 0 & 2mn \\ n^2 & m^2 & 0 & 0 & 0 & -2mn \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & -n & 0 \\ 0 & 0 & 0 & n & m & 0 \\ -mn & mn & 0 & 0 & 0 & (m^2 - n^2) \end{bmatrix} \quad (14)$$

and $m = \cos \theta$, $n = \sin \theta$. Similarly, global ply strains are obtained according to

$$\bar{\epsilon}_i = [T(\theta)]_{ij}^{\epsilon} \epsilon_j \quad (15)$$

in which the strain transformation matrix is given by

$$[T(\theta)]_{ij}^{\epsilon} = \begin{bmatrix} m^2 & n^2 & 0 & 0 & 0 & -mn \\ n^2 & m^2 & 0 & 0 & 0 & mn \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & -n & 0 \\ 0 & 0 & 0 & n & m & 0 \\ -2mn & 2mn & 0 & 0 & 0 & (m^2 - n^2) \end{bmatrix} \quad (16)$$

The lamina stress-strain constitutive relationship, defined in the global (x,y,z) laminate coordinate system, is written explicitly as

$$\bar{\sigma}_i = \bar{C}_{ij} \bar{\epsilon}_j \quad \text{for} \quad (i, j = 1, 2, 3, 4, 5, 6) \quad (17)$$

in which the bars denote that the quantities are defined in the global coordinate system.

Combining equations (5), (13), (15), and (17), it can be shown that the lamina stiffness matrix,

\bar{C}_{ij} , can be expressed explicitly in terms of the principal lamina stiffness matrix and the ply orientation angle, θ , through the following expression:

$$\bar{C}_{ij} = [T(\theta)]_{ij}^s C'_{ij} \{ [T(\theta)]_{ij}^{\epsilon} \}^{-1} \quad (18)$$

2.4 Effective Laminated Media Constitutive Relations

The theoretical background of a stress-strain constitutive model for predicting the effective material constants of laminated media is summarized in this section. The detailed theoretical development of this model is presented elsewhere (Chou, Carleone, and Hsu 1972), and therefore, only the most significant features are reviewed here.

First, an N layered laminate (i.e., representative sublaminar configuration), such as the one depicted in Figure 3, is identified. The "laminate" is considered to be representative of a "small element" (i.e., material point) in a larger structural body. The objective is to obtain a single constitutive relationship that can be used to define the effective (i.e., homogeneous) stress-strain response of the laminate. The following expression is used to represent the effective stress-strain constitutive relationship for the laminate

$$\bar{\sigma}_i^* = \bar{C}_{ij}^* \bar{\epsilon}_j^* \quad \text{for} \quad (i, j = 1, 2, 3, 4, 5, 6) \quad (19)$$

The superscript “*” is used here to denote the “average” or effective laminate stress and strain quantities.

In-plane strains are assumed to be uniform (i.e., constant within each ply) and equal to the effective strains of the laminate. Mathematically, this is expressed as

$$\bar{\epsilon}_i^k = \bar{\epsilon}_i^* \quad \text{for} \quad (i = 1, 2, 6; k = 1, 2, \dots, N) \quad (20)$$

in which $\bar{\epsilon}_i^k$ represents the strain in the k^{th} ply of the laminate (see ply-numbering convention in Figure 3).

To ensure stress continuity across ply interfaces, all ply stress components associated with the out-of-plane direction (i.e., z-direction) are assumed to be uniform and equal to the corresponding effective stresses in the laminate. Mathematically, this is expressed as

$$\bar{\sigma}_i^k = \bar{\sigma}_i^* \quad \text{for} \quad (i = 3, 4, 5; k = 1, 2, \dots, N) \quad (21)$$

in which $\bar{\sigma}_i^k$ represents the stress in the k^{th} ply of the laminate.

All remaining effective laminate stresses and strains are assumed to be the volume average of all their corresponding ply stress and strain components, respectively. Mathematically, these assumptions are expressed as

$$\bar{\epsilon}_i^* = \sum_{k=1}^N V^k \bar{\epsilon}_i^k \quad \text{for} \quad (i = 3, 4, 5) \quad (22)$$

$$\bar{\sigma}_i^* = \sum_{k=1}^N V^k \bar{\sigma}_i^k \quad \text{for} \quad (i = 1, 2, 6) \quad (23)$$

in which V^k is the ratio of the original (i.e., undeformed) volume of the k^{th} ply over the original volume of the entire laminate.

Equation (17) is re-written below using the superscript notation to emphasize that it applies to each individual ply in the laminate:

$$\bar{\sigma}_i^k = \bar{C}_{ij}^k \bar{\epsilon}_j^k \quad \text{for} \quad (i, j = 1, 2, 3, 4, 5, 6; k=1, 2, \dots, N) \quad (24)$$

Equations (19) through (24) represent $12N+6$ linear algebraic equations in $12N+12$ unknowns. Chou, Carleone, and Hsu (1972) found that the solution to Equations (19) through (24) yields the following effective three-dimensional stress-strain constitutive relation which can be used as an equivalent (i.e., homogeneous) representation for the laminated media where the coefficients in the laminate stiffness matrix, \bar{C}_{ij}^* , are given by

$$\bar{C}_{ij}^* = \sum_{k=1}^N V^k \left[\bar{C}_{ij}^k - \frac{\bar{C}_{13}^k \bar{C}_{3j}^k}{\bar{C}_{33}^k} + \frac{\bar{C}_{i3}^k \sum_{l=1}^N \frac{V^l \bar{C}_{3j}^k}{\bar{C}_{33}^k}}{\bar{C}_{33}^k \sum_{l=1}^N \frac{V^l}{\bar{C}_{33}^k}} \right] \text{ for } (i, j = 1, 2, 3, 6) \quad (25)$$

$$\bar{C}_{ij}^* = \bar{C}_{ji}^* = 0 \text{ for } (i = 1, 2, 3, 6; j = 4, 5) \quad (26)$$

and

$$\bar{C}_{ij}^* = \left[\frac{\sum_{k=1}^N \frac{V^k}{\Delta_k} \bar{C}_{ij}^k}{\sum_{k=1}^N \sum_{l=1}^N \frac{V^k V^l}{\Delta_k \Delta_l} (\bar{C}_{44}^k \bar{C}_{55}^k - \bar{C}_{45}^k \bar{C}_{54}^k)} \right] \text{ for } (i, j = 4, 5) \quad (27)$$

in which

$$\Delta_k = \begin{vmatrix} \bar{C}_{44}^k & \bar{C}_{45}^k \\ \bar{C}_{54}^k & \bar{C}_{55}^k \end{vmatrix} = \bar{C}_{44}^k \bar{C}_{55}^k - \bar{C}_{45}^k \bar{C}_{54}^k \quad (28)$$

The effective stress-strain constitutive relation for the laminated media is therefore given by Equations (19) and (25) through (28). The effective laminate stiffness matrix described is included in the LAM3D code predictions.

2.5 Effective Laminated Media Engineering Constants

In the following sections, the constitutive relation just presented will be used to define the effective thermo-mechanical properties (i.e., typical engineering constants) for the laminated media. These quantities are often viewed as a more convenient form of input for structural analyses purposes (e.g., finite element methods).

2.5.1 Mechanical Properties

A set of effective three-dimensional engineering constants, which can be used to represent the laminated media as a homogeneous continuum, is now presented. The engineering constants, defined in the global laminate coordinate system, include the extensional young's moduli, E_x , E_y , and E_z ; the shear moduli, G_{yz} , G_{xz} , and G_{xy} ; the major Poisson's ratios, ν_{yz} , ν_{xz} , ν_{xy} ; and the minor Poisson's ratios, ν_{zy} , ν_{zx} , ν_{yx} (see Figure 3 for coordinate reference). Note that the Poisson's ratios presented here are defined in accordance with typical conventions (i.e., $\nu_{ij} = -\epsilon_j / \epsilon_i$).

The constitutive relation equation (19) (which is in the "stiffness" form) is first re-expressed in its "compliance" form. Inverting both sides of equation (19), the effective laminate compliance relationship becomes

$$\bar{\epsilon}_i^* = \bar{H}_{ij}^* \bar{\sigma}_j^* \quad \text{for} \quad (i, j = 1, 2, 3, 4, 5, 6) \quad (29)$$

in which \bar{H}_{ij}^* is termed the laminate compliance matrix, simply the inverse of the laminate stiffness matrix (i.e., $\bar{H}_{ij}^* = \bar{C}_{ij}^{*-1}$). From this point, the effective mechanical engineering constants can be expressed explicitly in terms of the compliance coefficients, \bar{H}_{ij}^* . The effective extensional moduli are given by

$$E_x = 1/\bar{H}_{11}^* \quad (30)$$

$$E_y = 1/\bar{H}_{22}^* \quad (31)$$

and

$$E_z = 1/\bar{H}_{33}^* \quad (32)$$

The effective shear moduli are given by

$$G_{yz} = 1/\bar{H}_{44}^* \quad (33)$$

$$G_{xz} = 1/\bar{H}_{55}^* \quad (34)$$

and

$$G_{xy} = 1/\bar{H}_{66}^* \quad (35)$$

The effective major Poisson's ratios are given by

$$\nu_{yz} = -\bar{H}_{23}^* / \bar{H}_{22}^* \quad (36)$$

$$\nu_{xz} = -\bar{H}_{13}^* / \bar{H}_{11}^* \quad (37)$$

$$v_{xy} = - \bar{H}_{12}^* / \bar{H}_{11}^* \quad (38)$$

and the effective minor Poisson ratios are given by

$$v_{zy} = - \bar{H}_{23}^* / \bar{H}_{33}^* \quad (39)$$

$$v_{zx} = - \bar{H}_{13}^* / \bar{H}_{33}^* \quad (40)$$

and

$$v_{yx} = - \bar{H}_{12}^* / \bar{H}_{22}^* \quad (41)$$

The effective mechanical properties just described are included in the LAM3D code predictions.

2.5.2 Thermal Expansion Coefficients

Effective thermal expansion coefficients can also be derived for the laminated media. The principal direction thermal expansion coefficients for the individual lamina are first rotated in the global laminate coordinate system. The strain rotation matrix is used (see Equation (16)):

$$\bar{\alpha}_i = [T(\theta)]_{ij}^{\epsilon} \alpha_j' \quad (42)$$

in which, again, the bar notation denotes the laminate coordinate system and the prime denotes the principal material coordinate system. A quantity that essentially represents a non-dimensional (average) uniform thermal loading condition on the laminate is first computed according to

$$\bar{N}_i^{th*} = \sum_{k=1}^N \bar{C}_{ij}^k \bar{\alpha}_j^k V^k \quad \text{for } (i, j = 1, 2, 3, 4, 5, 6) \quad (43)$$

The effective laminated media thermal expansion coefficients, defined in the global laminate coordinate system, are then given by

$$\bar{\alpha}_x^* = \bar{H}_{1j}^* \bar{N}_j^{th*} \quad \text{for } (j = 1, \dots, 6) \quad (44)$$

$$\bar{\alpha}_y^* = \bar{H}_{2j}^* \bar{N}_j^{th*} \quad \text{for } (j = 1, \dots, 6) \quad (45)$$

$$\bar{\alpha}_z^* = \bar{H}_{3j}^* \bar{N}_j^{th*} \quad \text{for } (j = 1, \dots, 6) \quad (46)$$

$$\bar{\alpha}_{yz}^* = \bar{H}_{4j}^* \bar{N}_j^{th*} \quad \text{for } (j = 1, \dots, 6) \quad (47)$$

$$\bar{\alpha}_{xz}^* = \bar{H}_{5j}^* \bar{N}_j^{th*} \quad \text{for } (j = 1, \dots, 6) \quad (48)$$

and

$$\bar{\alpha}_{xy}^* = \bar{H}_{6j}^* \bar{N}_j^{th*} \quad \text{for } (j = 1, \dots, 6) \quad (49)$$

The effective laminate thermal expansion coefficients just described are included in the LAM3D code predictions.

2.6 Ply-Level Stress and Strain Calculations

To assess failure within the laminated media because of mechanical and/or thermal loading, all ply-level stress and strains values must be computed. The following sections describe this procedure.

2.6.1 Applied Mechanical Loading

The assumption here is that the applied mechanical loading on the laminated media ($\bar{\sigma}_i^*$) is known, uniform and represents the “average” or “effective” stress acting on the sub-laminate configuration. The associated “effective” or “smeared” laminate strains ($\bar{\epsilon}_i^*$) can be obtained directly from Equation (29). From the assumption made in Equation (20), all in-plane strain values (defined in the global (x, y, z) coordinate system) for Plies 1 through N are therefore known. Similarly, from the assumption made in Equation (21), all out-of-plane stresses for Plies 1 through N are known (also defined in the global (x, y, z) coordinate system). The out-of-plane ply strains and in-plane ply stresses remain to be determined.

Sun and Liao (1990) derived the following expression for determining the remaining out-of-plane ply strains:

$$\begin{bmatrix} \bar{\epsilon}_3^k \\ \bar{\epsilon}_4^k \\ \bar{\epsilon}_5^k \end{bmatrix} = \begin{bmatrix} \bar{C}_{33}^k & \bar{C}_{34}^k & \bar{C}_{35}^k \\ \bar{C}_{43}^k & \bar{C}_{44}^k & \bar{C}_{45}^k \\ \bar{C}_{53}^k & \bar{C}_{54}^k & \bar{C}_{55}^k \end{bmatrix}^{-1} \left[\begin{bmatrix} \bar{\sigma}_3^k \\ \bar{\sigma}_4^k \\ \bar{\sigma}_5^k \end{bmatrix} - \begin{bmatrix} \bar{C}_{31}^k & \bar{C}_{32}^k & \bar{C}_{36}^k \\ \bar{C}_{41}^k & \bar{C}_{42}^k & \bar{C}_{46}^k \\ \bar{C}_{51}^k & \bar{C}_{52}^k & \bar{C}_{56}^k \end{bmatrix} \begin{bmatrix} \bar{\epsilon}_1^k \\ \bar{\epsilon}_2^k \\ \bar{\epsilon}_6^k \end{bmatrix} \right] \quad (50)$$

Once all the ply strains are known, the remaining in-plane ply stresses can be calculated straightforwardly through the following relation:

$$\begin{bmatrix} \bar{\sigma}_1^k \\ \bar{\sigma}_2^k \\ \bar{\sigma}_6^k \end{bmatrix} = \begin{bmatrix} \bar{C}_{11}^k & \bar{C}_{12}^k & \bar{C}_{13}^k & \bar{C}_{14}^k & \bar{C}_{15}^k & \bar{C}_{16}^k \\ \bar{C}_{21}^k & \bar{C}_{22}^k & \bar{C}_{23}^k & \bar{C}_{24}^k & \bar{C}_{25}^k & \bar{C}_{26}^k \\ \bar{C}_{61}^k & \bar{C}_{62}^k & \bar{C}_{63}^k & \bar{C}_{64}^k & \bar{C}_{65}^k & \bar{C}_{66}^k \end{bmatrix} \begin{bmatrix} \bar{\epsilon}_1^k \\ \bar{\epsilon}_2^k \\ \bar{\epsilon}_3^k \\ \bar{\epsilon}_4^k \\ \bar{\epsilon}_5^k \\ \bar{\epsilon}_6^k \end{bmatrix} \quad (51)$$

The mechanical ply stress and strains just described are included in the LAM3D code predictions.

2.6.2 Thermal Loading

Ply stresses and ply strains resulting from thermal loading are independent of the mechanical ply stresses and strains but are computed in an analogous manner. Thermal ply stresses result from the fact that the individual plies within the laminate have different thermal expansion coefficients in different directions but are constrained to deform or strain uniformly. To determine thermal ply stresses and strains, the effective or average thermal stress for the laminated media is first computed for a given prescribed thermal loading. In this formulation, the thermal load for each ply in the laminated media (i.e., ΔT^k) can be different. The effective thermal stress acting on the laminate is given by

$$(\bar{\sigma}_i^{th})^* = \sum_{k=1}^N \bar{C}_{ij}^k \bar{\alpha}_j^k V^k \Delta T^k \quad \text{for} \quad (i, j = 1, 2, 3, 4, 5, 6) \quad (52)$$

The actual thermal strain or deformation that the laminate undergoes, $(\bar{\epsilon}_i^{th})^*$, is computed in a similar manner to the mechanical strains, through the following relationship:

$$(\bar{\epsilon}_i^{th})^* = \bar{H}_{ij}^* (\bar{\sigma}_j^{th})^* \quad \text{for} \quad (i, j = 1, 2, 3, 4, 5, 6) \quad (53)$$

As with the calculation of the mechanical ply strains, the in-plane thermal ply strains are assumed to be constant and equal to the actual in-plane thermal laminate strains:

$$(\bar{\epsilon}_i^{th})^k = (\bar{\epsilon}_i^{th})^* \quad \text{for} \quad (i = 1, 2, 6; k = 1, 2, \dots, N) \quad (54)$$

in which $(\bar{\epsilon}_i^{th})^k$ represents the thermal strain in the k^{th} ply. The out-of-plane thermal ply stresses are assumed to be constant and equal to the effective out-of-plane thermal laminate stress

$$(\bar{\sigma}_i^{th})^k = (\bar{\sigma}_i^{th})^* \quad \text{for} \quad (i = 3, 4, 5; k = 1, 2, \dots, N) \quad (55)$$

The remaining out-of-plane thermal ply strains are then straightforwardly computed through a relationship similar to that given in Equation (50):

$$\begin{bmatrix} (\bar{\epsilon}_3^{th})^k \\ (\bar{\epsilon}_4^{th})^k \\ (\bar{\epsilon}_5^{th})^k \end{bmatrix} = \begin{bmatrix} \bar{C}_{33}^k \bar{C}_{34}^k \bar{C}_{35}^k \\ \bar{C}_{43}^k \bar{C}_{44}^k \bar{C}_{45}^k \\ \bar{C}_{53}^k \bar{C}_{54}^k \bar{C}_{55}^k \end{bmatrix}^{-1} \left[\begin{bmatrix} (\bar{\sigma}_3^{th})^* \\ (\bar{\sigma}_4^{th})^* \\ (\bar{\sigma}_5^{th})^* \end{bmatrix} - \begin{bmatrix} \bar{C}_{31}^k \bar{C}_{32}^k \bar{C}_{36}^k \\ \bar{C}_{41}^k \bar{C}_{42}^k \bar{C}_{46}^k \\ \bar{C}_{51}^k \bar{C}_{52}^k \bar{C}_{56}^k \end{bmatrix} \begin{bmatrix} (\bar{\epsilon}_1^{th})^* \\ (\bar{\epsilon}_2^{th})^* \\ (\bar{\epsilon}_6^{th})^* \end{bmatrix} \right] \quad (56)$$

At this point, the stress-free expansional strains (i.e., the strains that would be induced in each ply if they were not constrained to deform together) are subtracted from the actual laminate thermal strains. The resulting thermal ply strains are actually those that will be used to compute the remaining, unknown, in-plane thermal ply stresses and are expressed according to

$$(\bar{\epsilon}_i^{th})^k = (\bar{\epsilon}_i^{th})^k - \bar{\alpha}_i^k \Delta T^k \quad \text{for} \quad (i = 1, 2, 3, 4, 5, 6; k=1,2, \dots, n) \quad (57)$$

The out-of-plane thermal ply stresses are given in Equation (55), and the remaining in-plane thermal ply stresses are given by

$$\begin{bmatrix} (\bar{\sigma}_1^{th})^k \\ (\bar{\sigma}_2^{th})^k \\ (\bar{\sigma}_6^{th})^k \end{bmatrix} = \begin{bmatrix} \bar{C}_{11}^k \bar{C}_{12}^k \bar{C}_{13}^k \bar{C}_{14}^k \bar{C}_{15}^k \bar{C}_{16}^k \\ \bar{C}_{21}^k \bar{C}_{22}^k \bar{C}_{23}^k \bar{C}_{24}^k \bar{C}_{25}^k \bar{C}_{26}^k \\ \bar{C}_{61}^k \bar{C}_{62}^k \bar{C}_{63}^k \bar{C}_{64}^k \bar{C}_{65}^k \bar{C}_{66}^k \end{bmatrix} \begin{bmatrix} (\bar{\epsilon}_1^{th})^k \\ (\bar{\epsilon}_2^{th})^k \\ (\bar{\epsilon}_3^{th})^k \\ (\bar{\epsilon}_4^{th})^k \\ (\bar{\epsilon}_5^{th})^k \\ (\bar{\epsilon}_6^{th})^k \end{bmatrix} \quad (58)$$

The thermal ply stress and strains just described are included in the LAM3D code predictions.

2.7 Laminated Media Strength Predictions

The overall failure assessment of the laminated media or laminate is based on the stress and/or strain states that exist in the individual plies as well as ply-level-based (lamina) composite failure criterion and some type of failure solution strategy. For a given set of mechanical and/or thermal loads imposed on the laminate, the individual ply stresses and strains are first calculated according to the procedure outlined in the previous sections. Any one of a number of lamina failure criteria can then be used to make a failure assessment in all the individual plies within the laminate. An overall laminate strength prediction is then made by considering the ply-level failure assessment of all plies in conjunction with some type of failure solution strategy. A failure solution strategy is simply an assumption or set of assumptions that define how load is redistributed from the failed plies to the other plies within the laminate.

In the following discussion, the specific lamina failure criteria used in LAM3D are described. Various failure solution strategies also employed in LAM3D are then presented (i.e., first ply and progressive ply failure solution strategies). The section closes by describing the manner in which overall laminate strength predictions (quantitative and qualitative laminate failure assessments) are presented by LAM3D.

2.7.1 *Lamina Failure Criteria*

Several articles review the failure theories for composite materials. Nahas (1986) gives an overview of failure theories for composites and isotropic materials proposed before 1986. Hahn and Kallas (1992) have presented a detailed analysis of several more recent composite failure theories. This report will briefly review the eight lamina failure criteria that have been included in the LAM3D code.

2.7.1.1 Von Mises-Hencky (isotropic materials only)

The Von Mises-Hencky or maximum distortion energy theory predicts that a ductile material under a general stress state will yield when its shear distortional energy (the total strain energy minus the strain energy attributable to change in volume) is equal to the shear distortional energy under simple tension (Hertzberg, 1989; Nahas, 1986). This yield theory is only valid for isotropic materials and, consequently, is not generally appropriate for composite materials. It has shown good correlation with test data for metals under multi-axial loading since it includes the interactive effects of all the stress components. The Von Mises-Hencky theory is applied by first calculating the equivalent or effective stress (σ_e) acting on a material element (Hertzberg, 1989) according to

$$\sigma_e = \frac{\sqrt{2}}{2} \left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2) \right]^{0.5} \quad (59)$$

This failure theory predicts that the material will fail when the effective stress becomes greater than the material yield strength, σ_y , or when

$$\sigma_e > \sigma_y. \quad (60)$$

2.7.1.2 Maximum Stress

Perhaps the most widely used failure criterion for unidirectional composites is the maximum stress failure criterion (Tsai, 1987; Nahas, 1986), which predicts that a material will fail when the magnitude of the stress in any direction exceeds its corresponding allowable level in that direction. This criterion is valid for both isotropic and anisotropic materials. However, it does not consider interactions between the various stress components and, therefore, has the potential to be inaccurate for multi-axial stress states. The most significant advantage of this failure criterion is that it identifies the specific mode of failure within a ply. Failure in any principal direction of the material is predicted when any of the following conditions exist:

$$\text{if } \sigma_1 > 0 \text{ and if } \sigma_1 > X1T, \text{ then the failure mode is fiber tension} \quad (61a)$$

$$\text{if } \sigma_1 < 0 \text{ and if } |\sigma_1| > X1C, \text{ then the failure mode is fiber compression} \quad (61b)$$

$$\text{if } \sigma_2 > 0 \text{ and if } \sigma_2 > X2T, \text{ then the failure mode is matrix tension} \quad (61c)$$

$$\text{if } \sigma_2 < 0 \text{ and if } |\sigma_2| > X2C, \text{ then the failure mode is matrix compression} \quad (61d)$$

$$\text{if } \sigma_3 > 0 \text{ and if } \sigma_3 > X3T, \text{ then the failure mode is matrix tension} \quad (61e)$$

$$\text{if } \sigma_3 < 0 \text{ and if } |\sigma_3| > X3C, \text{ then the failure mode is matrix compression} \quad (61f)$$

$$\text{if } |\sigma_4| > X23, \text{ then the failure mode is interlaminar shear} \quad (61e)$$

$$\text{if } |\sigma_5| > X13, \text{ then the failure mode is interlaminar shear} \quad (61f)$$

$$\text{if } |\sigma_6| > X12, \text{ then the failure mode is in-plane shear} \quad (61g)$$

In Equations (61a) through (61g), σ_1 through σ_6 are the six principal ply stresses, X1T is the tensile strength in the 1-direction (longitudinal), X1C is the compressive strength in the 1-direction, X2T is the tensile strength in the 2-direction (transverse), X2C is the compressive strength in the 2-direction, X3T is the tensile strength in the 3-direction, X3C is the compressive strength in the 3-direction, X23 is the shear strength in the 23-plane, X13 is the shear strength in the 13-plane, and X12 is the shear strength in the 12-plane.

2.7.1.3 Maximum Strain

The maximum strain failure criterion (Tsai, 1987; Nahas, 1986) predicts that a material will fail when the strain in any direction exceeds its allowable level. This failure criterion is similar to the maximum stress criterion, except that it accounts for some of the interactions between the stresses that are attributable to the Poisson's effects in the material (i.e., stresses in the 1- and 3- directions will affect the strain in the 2-direction). The failure criterion is applied in the exact same manner as the maximum stress failure criterion. The mechanical strains in the six directions (ϵ_1 , ϵ_2 , ϵ_3 , ϵ_4 , ϵ_5 , and ϵ_6) are compared to their corresponding maximum strain allowables ($Y1T$, $Y1C$, $Y2T$, $Y2C$, $Y3T$, $Y3C$, $Y23$, $Y13$, and $Y12$) in the same manner as described for the maximum stress criterion, for example,

if $\epsilon_1 > 0$ and if $\epsilon_1 > Y1T$, then the failure mode is fiber tension (62a)

if $\epsilon_1 < 0$ and if $|\epsilon_1| > Y1C$, then the failure mode is fiber compression (62b)

if $\epsilon_2 > 0$ and if $\epsilon_2 > Y2T$, then the failure mode is matrix tension (62c)

if $\epsilon_2 < 0$ and if $|\epsilon_2| > Y2C$, then the failure mode is matrix compression (62d)

if $\epsilon_3 > 0$ and if $\epsilon_3 > Y3T$, then the failure mode is matrix tension (62e)

if $\epsilon_3 < 0$ and if $|\epsilon_3| > Y3C$, then the failure mode is matrix compression (62f)

if $|\epsilon_4| > Y23$, then the failure mode is interlaminar shear (62e)

if $|\epsilon_5| > Y13$, then the failure mode is interlaminar shear (62f)

if $|\epsilon_6| > Y12$, then the failure mode is in-plane shear (62g)

2.7.1.4 Hydrostatic Pressure Adjusted

The hydrostatic pressure-adjusted failure criterion is actually a modified version of the maximum stress failure criterion that accounts for stress interactions under compressive loading. The criterion is based on empirical observations reported in the literature (Hahn and Kallas, 1992). The failure criterion essentially states that all the compression strength allowables in the principal directions ($X1C$, $X2C$, and $X3C$) and the shear strength allowables ($X23$, $X13$, and $X12$) increase with the hydrostatic pressure state (HP) that exists within the 23 planes of a ply. A review of this phenomenon is discussed in a separate report (Hoppel, Bogetti, Gillespie 1995).

For the particular version of this criterion presented in this report, several assumptions are made. First, the "hydrostatic pressure state" in the 23 planes referred to here (HP) is not that which is defined in the traditional sense. It is assumed to be equal to either (1) the minimum of $|\sigma_2|$ and $|\sigma_3|$ (when both are compressive), (2) the average of σ_2 and σ_3 (when this average is compressive), or (3) zero when either of these two conditions is not satisfied. In addition, tensile

strength allowables are assumed to be independent of this hydrostatic pressure influence, and the strength allowables in the 2- and 3-directions are assumed to be the equal (i.e., $X_{2T}=X_{3T}$, $X_{2C}=X_{3C}$). It is further assumed that all shear strength allowables are equal (i.e., $S_{23}=S_{13}=S_{12}$).

Material strength here is assumed to be a bi-linear function with the respect to the hydrostatic pressure state in the 23 planes of a ply. This relationship is expressed explicitly as a function of the assumed hydrostatic pressure state, HP, according to

$$X_{1C}(HP)=X_{1C}(0)+ML_1 \cdot HP \text{ for } (HP < LTP) \quad (63a)$$

and

$$X_{1C}(HP)=(X_{1C}(0)+ML_1 \cdot LTP)+ML_2 \cdot HP \text{ for } (HP > LTP) \quad (63b)$$

in which $X_{1C}(0)$ is the usual longitudinal compression strength allowable (i.e., $X_{1C}(0)=X_{1C}$); ML_1 and ML_2 are the two slopes describing the bi-linear relationship, and LTP is the hydrostatic pressure state in which a change in slope of the X_{1C} versus HP relationship occurs. Similarly, for the transverse directions,

$$X_{2C}(HP)=X_{3C}(HP)=X_{2C}(0)+MT_1 \cdot HP \text{ for } (HP < TTP) \quad (63c)$$

and

$$X_{2C}(HP)=X_{3C}(HP)=(X_{2C}(0)+MT_1 \cdot TTP)+MT_2 \cdot HP \text{ for } (HP > TTP) \quad (63d)$$

in which $X_{2C}(0)$ is the usual transverse compression strength allowable (i.e., $X_{2C}(0)=X_{2C}=X_{3C}$); MT_1 and MT_2 are the two slopes describing the bi-linear relationship, and TTP is the hydrostatic pressure state in which a change in slope of the X_{2C} (or X_{3C}) versus HP relationship occurs. For the shear directions, the following relationships are used:

$$X_{23}(HP)=X_{13}(HP)=X_{12}(HP)=S+MS_1 \cdot HP \text{ for } (HP < STP) \quad (63e)$$

and

$$X_{23}(HP)=X_{13}(HP)=X_{12}(HP)=(S+MS_1 \cdot STP)+MS_2 \cdot HP \text{ for } (HP > STP) \quad (63f)$$

in which S is the usual shear strength allowable (i.e., $S=X_{23}=X_{13}=X_{12}$); MS_1 and MS_2 are the two slopes describing the bi-linear relationship, and STP is the hydrostatic pressure state in which a change in slope of the S (or S_{23} or S_{13} or S_{12}) versus HP relationship occurs.

2.7.1.5 Tsai-Wu Quadratic Interaction

The Tsai-Wu quadratic interaction or tensor polynomial failure criterion (Tsai and Wu 1971) accounts for the interactive effects of a multi-axial stress state. Failure is predicted when the following condition occurs:

$$F_1\sigma_1 + F_2(\sigma_2 + \sigma_3) + F_{11}\sigma_1^2 + F_{22}(\sigma_2^2 + \sigma_3^2) + 2F_{12}\sigma_1(\sigma_2 + \sigma_3) + 2F_{23}\sigma_2\sigma_3 + 2F_{44}\sigma_4^2 + F_{66}(\sigma_5^2 + \sigma_6^2) \geq 1 \quad (64)$$

in which σ_1 through σ_6 are the principal stresses in the lamina. The constants $F_1, F_2, F_{11}, F_{22}, F_{12}, F_{23}, F_{44}$, and F_{66} are defined by the following expressions:

$$F_1 = \frac{1}{X_{1T}} - \frac{1}{X_{1C}} \quad (65a)$$

$$F_2 = \frac{1}{X_{2T}} - \frac{1}{X_{2C}} \quad (65b)$$

$$F_{11} = \frac{1}{(X_{1T})(X_{1C})} \quad (65c)$$

$$F_{22} = \frac{1}{(X_{2T})(X_{2C})} \quad (65d)$$

$$F_{44} = 2(F_{22} - F_{23}) \quad (65e)$$

$$F_{66} = \frac{1}{S_{23}^2} = \frac{1}{S_{13}^2} = \frac{1}{S_{12}^2} \quad (65f)$$

The constants F_{12} and F_{23} are determined experimentally. Methods to determine these constants are described in Tsai and Wu (1971) and Jiang and Tennyson (1989). This theory assumes that the material is transversely isotropic in the principal 1-2 plane of the composite. The major drawback of this failure criterion is that it does not distinguish among the various potential modes of failure.

2.7.1.6 Christensen's Criterion

Christensen (1988) proposed a strain-based failure criterion, which identifies failure as being either fiber dominated or matrix dominated while considering the multi-axial stress state for matrix-dominated failure. This criterion identifies three distinct failure modes for a composite lamina: fiber tension, fiber compression, and matrix failure. Christensen's failure criterion has been translated into a stress-based failure criterion by Hahn and Kallas (1992), and the stress-based failure criterion is employed in this work.

Fiber tension and fiber compression, respectively, are predicted to occur when either Equation 66a or 66b is satisfied.

$$\left| \frac{(Y1T)(E_1)}{\sigma_1 - \nu_{12}\sigma_2 - \nu_{13}\sigma_3} \right| \leq 1 \text{ (if } \sigma_1 - \nu_{12}\sigma_2 - \nu_{13}\sigma_3 > 0) \quad (66a)$$

$$\left| \frac{(Y1C)(E_1)}{\sigma_1 - \nu_{12}\sigma_2 - \nu_{13}\sigma_3} \right| \leq 1 \text{ (if } \sigma_1 - \nu_{12}\sigma_2 - \nu_{13}\sigma_3 < 0) \quad (66b)$$

in which Y1T is the tensile failure strain for the lamina in the 1 direction, Y1C is the compressive failure strain for the lamina in the 1 direction, and E_1 is the elastic Young's modulus in the fiber direction. Matrix-dominated failure is predicted to occur when the following condition exists:

$$A\sigma_1 + B(\sigma_2 + \sigma_3) + C\sigma_1^2 + D(\sigma_2^2 + \sigma_3^2) + E\sigma_1(\sigma_2 + \sigma_3) + F\sigma_2\sigma_3 + G\sigma_1^2 + H(\sigma_2^2 + \sigma_3^2) \leq 1 \quad (67)$$

in which the coefficients A, B, C, D, E, F, G, and H are given by

$$A = \frac{\alpha(1 - 2\nu_{12})}{K^2 E_1} \quad (68a)$$

$$B = \frac{\alpha(1 - \nu_{21} - \nu_{23})}{K^2 E_2} \quad (68b)$$

$$C = \frac{2(1 + \nu_{12})^2}{3K^2 E_1^2} \quad (68c)$$

$$D = \frac{2}{3K^2 E_2^2} [(1 + \nu_{21} + \nu_{21}^2) + (1 - \nu_{21})\nu_{23} + \nu_{23}^2] \quad (68d)$$

$$E = \frac{2}{3K^2 E_1 E_2} [(-1 - 2\nu_{21} + \nu_{23})(1 + \nu_{12})] \quad (68e)$$

$$F = \frac{2}{3K^2 E_2^2} [(-1 + 2\nu_{21} + 2\nu_{21}^2) - 2(2 + \nu_{21})\nu_{23} - \nu_{23}^2] \quad (68f)$$

$$G = \frac{1}{2K^2 G_{23}^2} \quad (68g)$$

$$H = \frac{1}{2K^2 G_{12}^2} \quad (68h)$$

The constants K and α are experimentally determined material parameters, E_1 and E_2 are the elastic moduli in the principal and transverse directions, G_{12} and G_{23} are the in-plane and out-of-plane shear moduli, respectively, and the ν_{ij} 's are the usual Poisson's ratios for the lamina.

2.7.1.7 Feng's Failure Criterion

Feng (1991) also proposed a strain-based failure criterion that differentiates between fiber-dominated and matrix-dominated failure under multi-axial loading. This criterion determines failure, based on the strain invariants in the lamina. Matrix-dominated failure is predicted to occur when the following relation exists:

$$A_1 J_1 + A_{11} J_1^2 + A_2 J_2 - 1 \geq 0 \quad (69)$$

in which A_1 , A_{11} , and A_2 are empirically determined parameters and J_1 and J_2 are the strain invariants given by

$$J_1 = \epsilon_1 + \epsilon_2 + \epsilon_3 \quad (70a)$$

$$J_2 = \{[(\epsilon_1 - \epsilon_2)^2 + (\epsilon_3 - \epsilon_2)^2 + (\epsilon_1 - \epsilon_3)^2] / 6\} + \epsilon_4^2 + \epsilon_5^2 + \epsilon_6^2 \quad (70b)$$

in which ϵ_1 , ϵ_2 , ϵ_3 , ϵ_4 , ϵ_5 , and ϵ_6 are the ply strains. Fiber-dominated failure is predicted to occur when the following is satisfied:

$$A_5 J_5 + A_{55} J_5^2 + A_4 J_4 - 1 \geq 0 \quad (71)$$

in which A_5 , A_{55} , and A_4 are experimentally determined parameters, and J_4 and J_5 are the strain invariants given by

$$J_4 = \epsilon_4^2 + \epsilon_5^2 \quad (72a)$$

$$J_5 = \epsilon_1 \quad (72b)$$

2.7.1.8 Modified Hashin's Failure Criterion

Hashin (1980) proposed a stress-based failure criterion for composite materials, which considers the tri-axial stress state for matrix failure modes but only considers the uni-axial stress state in the fiber direction for fiber-dominated failure modes. In the fiber direction, this criterion is the same as the maximum stress failure criterion. Gipple, Nuismer, and Camponeschi (1995) suggested modifications of the failure criterion proposed by Hashin in which they assume that compressive matrix failure occurs because of a shearing mechanism rather than through compression. The modified Hashin's failure criterion is currently being used by several government agencies and contractors and is presented in this report.

For the fiber-dominated failure modes, the modified Hashin failure criterion distinguishes between tension and compression according to the following conditions:

if $\sigma_1 > 0$ and if $\sigma_1 \geq X1T$, then the failure mode is fiber tension (73a)

if $\sigma_1 < 0$ and if $|\sigma_1| \geq X1C$, then the failure mode is fiber compression (73b)

Matrix-dominated failure is predicted when either of the following two conditions exists:

$$\left(\frac{\sigma_{nn}}{X2T}\right)^2 + \left(\frac{\sigma_{nt}}{X2C}\right)^2 + \left(\frac{\sigma_{nl}}{X12}\right)^2 \geq 1 \text{ (for } \sigma_{nn} > 0 \text{ (matrix tension))} \quad (74a)$$

$$\left(\frac{\sigma_{nt}}{X2C}\right)^2 + \left(\frac{\sigma_{nl}}{X12}\right)^2 \geq 1 \text{ (for } \sigma_{nn} < 0 \text{ (matrix compression))} \quad (74b)$$

in which the normal (σ_{nn}), normal-transverse (σ_{nt}), and normal-longitudinal (σ_{nl}) stresses are evaluated according to the following:

$$\sigma_{nn} = \left(\frac{\sigma_2 + \sigma_3}{2}\right) + \left(\frac{\sigma_2 - \sigma_3}{2}\right)\cos(2\beta) + \sigma_{23}\sin(2\beta) \quad (75a)$$

$$\sigma_{nt} = -\left(\frac{\sigma_2 - \sigma_3}{2}\right)\sin(2\beta) + \sigma_{23}\cos(2\beta) \quad (75b)$$

$$\sigma_{nl} = \sigma_{13}\sin\beta + \sigma_{12}\cos\beta \quad (75c)$$

Here, the angle β defines an orientation in the 2-3 plane in which the maximum matrix stress state in the lamina exists.

2.7.2 First Ply and Progressive Ply Failure Theories

While the failure assessment of a single ply or lamina is clearly defined by the particular failure criterion used, the strength or failure assessment of the overall laminate is somewhat less straightforward. For example, for a given set of mechanical and/or thermal loading imposed on the laminate, any number of various failure modes could occur in one or more plies. A single failure or multiple failures on the ply level do not necessarily result in catastrophic failure of the laminate. In fact, depending on the laminate stacking sequence and loading, a laminate will often undergo several minor ply-level failures before catastrophic failure in the laminate occurs.

The term first ply failure refers to an approach wherein the overall laminate strength is determined by the lowest point of laminate load that causes any failure mode to occur in any ply within the laminate. Although this approach is not uncommon, it generally provides laminate strength predictions that are far too conservative.

A more practical, and often more accurate approach is referred to as a progressive ply failure theory. In this situation, the laminate load is increased to a point when failure is first predicted in any mode within a ply within the laminate (first ply failure). This load level is noted. The corresponding lamina stiffness or stiffnesses for that ply are reduced to an insignificant value and the laminate is re-loaded until the next failure is detected. This allows ply-level stresses to redistribute within the laminate and simultaneously prevents stress levels from increasing in the previously detected failure modes of failure plies. The procedure of loading to failure and reducing corresponding stiffnesses is continued in an iterative manner until the laminate can no longer support the intended applied mechanical loading. Iterations are discontinued when laminate strains reach some predefined, arbitrarily large value (representing laminate failure). The ultimate laminate strength is defined as the largest load level reached during the loading strategy. This load level does not necessarily have to be the last load level reached during the solution strategy. In fact, load levels rise and fall during the iterative solution strategy as they are dictated by the complex interactions of load transfer between the different modes of all the plies within the laminate. The particular mode of failure and the actual ply that corresponds to the largest load level reached during the solution strategy are identified as the critical mode and critical ply responsible for ultimate laminate failure.

2.7.3 Failure Assessment Summary

Laminate strength predictions can be based on first ply or progressive ply failure analyses. For either approach, an ultimate load level can be predicted as a measure of the quantitative performance of the laminate. It is often desirable to describe the ultimate laminate load or laminate strength in terms of a safety factor (SF). Such a safety factor is defined as the ultimate load that a laminate could sustain (as defined by first ply or progressive ply failure) divided by the applied loading on the laminate. This ultimate load would, of course, be some scalar multiple of the applied three-dimensional loading. According to this definition, safety factors less than one would represent laminate failures.

A qualitative description of failure is also very useful in understanding laminate strength. Corresponding to the ultimate load predicted for the laminate, a critical mode of failure and a critical ply can be defined. This information is important since it facilitates the redesign or optimization of laminate architectures for particular loading requirements. The LAM3D code generates laminate strength predictions in terms of safety factor and provides critical failure mode and critical ply information.

2.8 LAM3D: A Three-Dimensional Laminated Composite Media Analysis Software Program

2.8.1 *Program Options*

The theory that has been described in this report has been codified into a user-friendly FORTRAN-based computer program called LAM3D. This section describes, in detail, the input and output format specifically for version 3.0 of LAM3D. The code is intended to be used for three-dimensional laminated media analysis and calculates (1) effective properties, (2) ultimate strengths under mechanical and/or thermal loading, and (3) the effective stress versus strain response of the laminate because of mechanical loading under progressive ply failure. The laminate may be composed of different types of ply materials (e.g., hybrid), and a non-uniform thermal loading may be superimposed on the laminate before mechanical loading. The progressive ply failure analysis is incremental in loading. At every point where ply-level failure is detected during the loading history, any or all of the effective laminate or ply-level details (properties, stresses, and strains) are calculated and are available for examination (output). Failure analysis information available for output includes the safety factor, the critical ply, and the associated failure mode.

Any self-consistent set of units may be used in the LAM3D (version 3.0) analysis. Typically for SI units, pascals (Pa) are used for pressure, meters (m) for distance, kilograms per meter cubed (kg/m^3) for density, and degrees Celsius ($^{\circ}\text{C}$) for temperature. For English units, pounds per square inch (lb/in^2) are used for pressure, inches (in.) for distance, slugs per inch cubed (slug/in^3) for density, and degrees Fahrenheit ($^{\circ}\text{F}$) for temperature.

The program LAM3D (version 3.0) is executed in an interactive mode and uses a convenient data base format for conducting parametric studies. Details of the LAM3D (version 3.0) program execution are described in a subsequent section.

2.8.2 *Input Summary*

2.8.2.1 Material Data Base

The LAM3D (version 3.0) data base input file is used to define the laminate to be analyzed. The sample data base presented in Appendix A will be used to illustrate the data base file format. In the data base, the data are separated by commas and spaces on each line. The exclamation point (!) represents a comment statement; all the information to the left of the exclamation point is data which are read directly as input into the program; the information to the right is ignored by the program. The data base is subdivided into three sections: the first consists of three

header cards, the second describes laminate details, and the third is essentially a "material library."

2.8.2.1.1 Section 1: Header Cards

The first and second header cards are set to the number 1. The third header card defines the number of different types of materials that are listed in the data base file. (In this example, this card is set to 3, since three materials are listed: aluminum, IM7-graphite/8551-epoxy, and S2-glass/3501-epoxy.)

2.8.2.1.2 Section 2: Laminate Architecture

The second section of the data base defines several important parameters for the laminate analysis. The first card of this section contains a region identification number (REG_ID), the failure criterion identification number (FAIL_CRT), the maximum number of iterations for progressive failure (ITERS), and a text parameter flag (ITEXT). The region identification number (REG_ID) is always set to the number 1. The failure criterion identification number (FAIL_CRT) defines the failure criterion that will be used for all the plies in the laminate. A value between 1.0 and 8.0 is required with (1.0) corresponding to the Von Mises-Hencky failure criterion, (2.0) to the maximum stress failure criterion, (3.0) to the maximum strain failure criterion, (4.0) to the hydrostatic pressure-adjusted failure criterion, (5.0) to the Tsai-Wu quadratic interaction failure criterion, (6.0) to Christensen's failure criterion, (7.0) to Feng's failure criterion, and (8.0) to the modified Hashin failure criterion. The number of iterations (ITERS) is used to define the maximum number of times the program will allow the laminate to be loaded during the progressive ply failure analysis (a value of five is usually sufficient). The text parameter flag (ITEXT) is reserved to accommodate future enhancements of the LAM3D (version 3.0) code. It should, however, be set equal to zero.

The second card in this section contains three orientation parameters: (BETA), (PHI), and (SI). These parameters are reserved to accommodate future enhancements of the LAM3D (version 3.0) code and should all be set equal to zero. All spaces designated as (---) throughout the data base file are also reserved to accommodate future enhancements of the LAM3D (version 3.0) code and should all be set equal to zero. Consequently, the fourth parameter on this line should be set equal to zero.

The third card defines the number of plies in the laminate (NPLY) and the region type of the laminate (REG_TYP). The parameter (NPLY) defines the number of input cards (lines) that LAM3D (version 3.0) expects to read before it reads input from the material library section.

The region type parameter (REG_TYP) is reserved to accommodate future enhancements and should always be set to 1.0. Note that the total number of cards in this section will be equal to (NPLY+3).

The following (NPLY) number of cards are used to define the individual ply materials (MAT_ID), thicknesses (THICKNESS), orientation angles in degrees (ANGLE) and thermal ply loading (TEMP). The material identification number (MAT_ID) specifies the ply material type (defined in the material library section of the database) for each ply. (Note that by specifying the material type for each ply individually, the analysis of hybrid laminates is easily accommodated.) The thickness parameter (THICKNESS) is the relative thickness of the ply in the laminate. The ply angle (ANGLE) is the orientation between the longitudinal (1-direction) axis of the ply and the longitudinal axis (x-direction) of the laminate (see Figure 3). The thermal ply loading (TEMP) defines the difference between the operating temperature and processing temperature of the ply. (Note that by specifying the thermal ply loading in this manner, the laminate is not restricted to a uniform thermal loading profile.)

2.8.2.1.3 Section 3: Material Library

The last section of the data base file is structured in what could be considered a "material library." This section contains the three-dimensional ply properties, densities, and strength parameters for all the materials included in the data base. The number of materials listed in this section should be equal to that defined in the third card of the first section. Each individual material listed in the data base file occupies 45 cards (lines), with three columns (entries) for each card. The 45-card format for one material is described next. (Additional materials included in the data base file would be inserted in exactly the same format.)

The first card contains the material identification number (MAT_ID), the material type (MAT_TYPE), and the material density (MAT_DENS). The first material listed in the file would have an identification number (MAT_ID) of 1.0, the second 2.0, and so on. The material type parameter (MAT_TYPE) is reserved to accommodate future enhancements and should always be set to 1.0. The material density (MAT_DENS) should also be specified for the material.

The second card contains the elastic moduli for the material defined in its three principal coordinate directions (i.e., E_1 , E_2 , E_3). (Note that if the material is isotropic, the same value should be entered for all three moduli.) The third card contains the three major Poisson's ratios for the material (i.e., ν_{23} , ν_{13} , ν_{12}). The fourth card contains the three shear moduli for the

material (i.e., G_{23} , G_{13} , G_{12}). The fifth card contains the thermal expansion coefficients in the three principal coordinate directions (i.e., α_{11} , α_{22} , α_{33}).

Cards 6 through 45 contain all the parameters required to define the failure criteria of the material. For each failure criteria (1 through 8), the data base reserves five cards (lines) each with three columns. Since there are eight failure criteria, 40 cards are reserved for each material to define all the potentially required failure parameters.

The Von Mises failure criterion is the first one listed. The only datum the program requires for this failure criterion is the material yield strength, (σ_y), which should be listed in the first column of the first card (designated as VM1 in the sample data base file in Appendix A). Since the program does not require any additional information for this failure criterion, the remaining 14 positions should all be defined as 0.0. (Again, see the aluminum material in the data base file in Appendix A.)

The second failure criterion listed in the data base file is maximum stress, which requires nine constants. The nine constants (X_{1T} , X_{1C} , X_{2T} , X_{2C} , X_{3T} , X_{3C} , X_{23} , X_{13} , and X_{12} , defined in Section 2.7.1.2) are input in the format indicated by the IM7-graphite/8551-epoxy material ($MAT_ID=2.0$) entry listing in the example data base file.

The third failure criterion is maximum strain failure, which also requires nine constants. The constants (Y_{1T} , Y_{1C} , Y_{2T} , Y_{2C} , Y_{3T} , Y_{3C} , Y_{23} , Y_{13} , and Y_{12} , defined in Section 2.7.1.3) are input in the format indicated by the IM7-graphite/8551-epoxy material ($MAT_ID=2.0$) entry listing in the example data base file.

The fourth failure criterion is hydrostatic pressure-adjusted failure, which requires 15 constants. The constants required are defined in Section 2.7.1.4 and their format is indicated by the IM7-graphite/8551-epoxy material ($MAT_ID=2.0$) entry listing in the example data base file. The variable HPD is used to specify the manner in which the hydrostatic pressure state is calculated. For $HPD=1.0$, the hydrostatic pressure state is assumed to be equal to the minimum of $|\sigma_2|$ and $|\sigma_3|$ (when they are both compressive) and zero if either σ_2 or σ_3 are tensile. For $HPD=2.0$, the hydrostatic pressure state is assumed to be equal to the average of σ_2 and σ_3 (when this average is compressive) and zero if this average is tensile.

The fifth failure criterion is the Tsai-Wu quadratic interaction, which requires seven parameters to be input into the data base. The constants required are defined in Section 2.7.1.5

and their format is indicated by the IM7-graphite/8551-epoxy material (MAT_ID=2.0) entry listing in the example data base file.

Christensen's failure criterion is the sixth listed, and it requires 13 parameter entries in the data base file. The parameters E_1 , E_2 , E_3 , ν_{23} , ν_{13} , ν_{12} , G_{23} , G_{13} , and G_{12} are the usual ply properties. The parameters K , α (ALPHA), $Y1T$ and $Y1C$ are defined in Section 2.7.1.6 and their format is indicated by the IM7-graphite/8551-epoxy material (MAT_ID=2.0) entry listing in the example data base file.

The seventh failure criterion is Feng's, which requires the following six parameters: A_1 , A_{11} , A_2 , A_4 , A_5 , and A_{55} (see Section 2.7.1.7). Again, the format for these parameters is indicated by the IM7-graphite/8551-epoxy material (MAT_ID=2.0) entry listing in the example data base file.

The eighth failure criterion, modified Hashin's failure, requires the same nine parameters as the maximum stress failure criterion ($X1T$, $X2T$, $X3T$, $X1C$, $X2C$, $X3C$, $X23$, $X13$, and $X12$). The same format for entry into the database file as the maximum stress criterion is also used. (See the IM7-graphite/8551-epoxy material (MAT_ID=2.0) entry listing in the example data base.)

2.8.2.2 LAM3D (version 3.0) Program Execution

Execution of LAM3D is done in an interactive mode. The following section details the interactive questions that are encountered in a typical program run.

After appropriate compilation and execution commands are performed, the first required user input for LAM3D is entered to define the desired output to be written (stored) to an output file. The following text will be printed.

ENTER (1) YES OR (0) NO FOR THE FOLLOWING OUTPUT
IN SEQUENCE ON ONE LINE OF INPUT ...

- (1) OUTPUT OF INPUT
- (2) CIJ MATRICES
- (3) EFFECTIVE PROPERTIES
- (4) LOADING
- (5) MECHANICAL PLY STRESS AND STRAIN
- (6) THERMAL PLY STRESS AND STRAIN
- (7) FAILURE ANALYSIS DETAILS

For each of the seven items on the list, the user should enter a one (1) if he or she wants those data to be included in the program output or a zero (0) if those data are not to be included in the output. For example, to display all the information offered in the output, type

1, 1, 1, 1, 1, 1, 1 [CR]

in which [CR] is the carriage return. LAM3D will then ask the user what type of lamina constitutive model should be assumed (see Section 2.2 for model details):

ENTER TYPE OF LAMINA CONSTITUTIVE MODEL >

- (1) ORTHOTROPIC (9 I.E.C.)
- (2) TRANSVERSELY ISOTROPIC (5 I.E.C.)
- (3) IN-PLANE MODEL (3 I.E.C.)

In this example problem, the orthotropic lamina constitutive model will be used, so the user should respond by typing

1 [CR]

LAM3D will then ask the user what type of laminate constitutive model will be used (see Enie and Rizzo [1970] for simplified model and Section 2.4 or Chou, Carleone, and Hsu [1972] for exact model details):

ENTER TYPE OF LAMINATE CONSTITUTIVE MODEL >

- (1) SIMPLIFIED (RIZZO, 1970)
- (2) EXACT (CHOU, 1972)

In this example problem, the exact laminate constitutive model will be employed, so the user should respond by typing

2 [CR]

LAM3D will then ask the user to enter the mechanical loading (three-dimensional stresses) to be applied on the laminate in the x, y, and z directions in units of pounds per square inch:

ENTER AVERAGE STRESS X, Y, Z (lb/in²) >

The following response would indicate that a load of -1000 lb/in² was applied in the x-direction:

-1000, 0, 0 [CR]

LAM3D will then ask the user to enter the applied shear stresses on the laminate in the yz-, xz-, and xy-directions in units of pounds per square inch:

ENTER AVERAGE SHEAR STRESSES YZ, XZ, XY (lb/in²) >

The following response would indicate an applied load of 10 lb/in² in the yz-direction, 35 lb/in² in the xz-direction, and 10 lb/in² in the xy-direction:

10, 35, 10 [CR]

(Recall that all thermal loads are defined through the data base file [see Section 2.8.2.1].) With the following prompt, the name of the pre-established input data base file is entered:

ENTER NAME OF DATA BASE FILE >
dbase.in [CR]

Finally, LAM3D will ask the user to enter the name of the file where all requested output will be stored:

ENTER NAME OF OUTPUT FILE >
out [CR]

After this input is entered, the program will begin to run. LAM3D will write various comments on the screen as it is reading from the material data base and while it is computing. LAM3D will also write the user-requested information during execution to the indicated output file. During the failure analysis, if the degradation of the laminate because of progressive ply failure is detected, the program will report the following:

LOCAL LAMINATE STRAIN EXCEEDS TOLERANCE . . .

In addition, if the initially applied thermal stresses in the laminate cause failure (in any ply), the program will report the following:

THERMAL LOADS EXCEED ALLOWABLE FOR STRESS . . .
PROGRAM TERMINATED . . .
PLEASE RESTART WITH REDUCED THERMAL LOADING . . .

The output file format generated by LAM3D is discussed in the next section.

2.8.3 Output File Format Summary

A sample output file produced by LAM3D (generated by the input described in the previous sections of this report) is shown in Appendix B. The output file can be used to store a rather extensive amount of information related to the laminate analysis. For illustrative purposes, all the output options have been selected for this example. However, since all this information is generally not required, the output requests will generally be significantly reduced. First, output written to the file is a summary of the input that was used to define the laminate analysis. This includes information that is both user input and data base input (e.g., type of analysis options, ply properties, failure criteria, loading, etc.).

As the failure analysis is being executed, the program will report the requested detailed information about the laminate that is "changing" because of the progressive ply failures. For example, after each iteration in the failure analysis, the following (if all are requested) are written as output: the C_{ij} ply matrices in the principal ply coordinate system, the C_{ij} ply matrices in the global laminate coordinate system, the effective C_{ij} stiffness matrix for the laminate, the effective engineering constants for the laminate, the applied mechanical laminate stresses, the applied thermal ply loading, the effective mechanical laminate strains, the effective thermal laminate strains, the mechanical and thermal ply stresses in both the principal ply and global laminate coordinate systems, and a detailed description of the failure assessment for that iteration in the failure analysis. The failure assessment at each iteration gives a cumulative summary of the ply failure modes that have occurred as well as the associated safety factor for the iteration.

3. RESULTS AND DISCUSSION

3.1. Lamina Properties and Strength Allowables

In this section, LAM3D predictions for the effective thermo-mechanical laminate properties are validated against results presented in the published literature. Laminate properties and strength predictions are also correlated against experimental data. In these correlations, three different types of unidirectional composite material systems are considered: AS4/3501, IM7/PEEK, and IM9/8551. The three-dimensional unidirectional lamina properties and strength allowables used in the LAM3D predictions are summarized in Tables 1 and 2, respectively. The AS4/3501 lamina properties were taken from the published literature (U.S. Army Materials Technology Laboratory 1992). The unidirectional lamina properties and strength allowables used for the IM7/PEEK and IM9/8551 material systems were generated under a separate characterization effort (Gillespie 1994).

Table 1

Unidirectional Composite Lamina Properties Used in LAM3D Predictions
(E and G in units of MSI)

	AS4/3501 (MIL-HDBK-17, 1992)	IM7/PEEK (Gillespie 1994)	IM9/8551 (Gillespie 1994)
E ₁	16.5	22.19 ± 2.83	22.80 ± 2.65
E ₂	1.40	1.19 ± 0.14	1.23 ± 0.08
E ₃	1.40	1.19 ± 0.14	1.23 ± 0.08
v ₂₃	0.54	0.45	0.45
v ₁₃	0.33	0.33	0.33
v ₁₂	0.33	0.33	0.33
G ₂₃	0.45	0.41 ± 0.05	0.47 ± 0.04
G ₁₃	0.87	1.45 ± 0.09	1.37 ± 0.11
G ₁₂	0.87	1.45 ± 0.09	1.37 ± 0.11

Table 2

Composite Lamina Strength Allowables Used in LAM3D Predictions
(allowables in units of KSI)

	IM7/PEEK (Gillespie 1994)	IM9/8551 (Gillespie 1994)
X1T	2611.17 ± 7.87	415.05 ± 19.85
X2T	10.43 ± 1.16	9.76 ± 0.19
X3T	10.43 ± 1.16	9.76 ± 0.19
X1C	138.15 ± 6.88	147.26 ± 12.89
X2C	30.58 ± 2.68	28.98 ± 1.26
X3C	30.58 ± 2.68	28.98 ± 1.26
S23	10.90 ± 0.42	24.97 ± 0.23
S13	10.90 ± 0.42	24.97 ± 0.23
S12	10.90 ± 0.42	24.97 ± 0.23

3.2 Effective Thermo-Mechanical Laminate Property Predictions

Effective thermo-mechanical laminate properties generated from the LAM3D code have been validated against other theoretical predictions in the published literature for some selected laminate architectures. Correlation of the effective laminate mechanical properties are presented next. It is noted that, although not presented here, the effective laminate thermal expansion coefficients have been validated against predictions reported by Pagano (1974c).

Theoretical predictions of the effective mechanical engineering constants for several AS4/3501 composite laminate architectures by Pagano (1974a), Sun and Li (1988), Roy and Tsai (1992) and classical laminated plate theory (CLPT) (Whitney 1987) are compared with LAM3D predictions (Chou, Carleone, & Hsu 1972). In Table 3, the comparison of predictions for the following laminate architectures is made: $[0/0/90/90/0/0]$, $[0/90/+45/-45/-45/+45/90/0]$ and $[+30/-30/-30/+30]$. Excellent agreement between the predictions gives indication that the LAM3D predictions are accurate and consistent with the proposed theory of (Chou, Carleone, & Hsu 1972).

Correlation of LAM3D effective mechanical property predictions are also made against the experimental results of Gillespie (1994) for a $[0/0/-45/+45/+45/-45/0/0]$ laminate architecture in both the IM7/PEEK and IM9/8551 materials. Reasonable agreement between the measured effective laminate moduli and the LAM3D predictions is presented in Table 4 for both the x- and y-directions.

3.3 Ultimate Laminate Strength Predictions

Uni-axial strengths of the $[0/0/-45/+45/+45/-45/0/0]$ laminate architecture in both the IM7/PEEK and IM9/8551 material systems were measured in tension and compression in both the x- and y-directions. LAM3D predictions for each material and load case were generated for comparison. The maximum stress lamina failure criterion was assumed in all the predictions presented in this correlation. The progressive ply failure theory discussed in Section 2.7.2 was also implemented.

Table 3

Predicted Effective 3D Properties for Various AS4/3501-7 Laminates
(E and G in units of MSI)

	CLPT (Whitney 1987)	Pagano (Pagano 1974)	Sun (Sun and Li 1988)	Roy (Roy and Tsai 1992)	Chou (Chou et al. 1972)
[0/0/90/90/0/0] Laminate Properties					
E _x	11.5	11.5	11.5	11.5	11.5
E _y	6.48	6.47	6.47	6.47	6.47
E _z	---	1.80	1.80	1.65	1.80
v _{yz}	---	.519	.520	.465	.519
v _{xz}	---	.488	.489	.402	.488
v _{xy}	.073	.073	.074	.072	.073
G _{yz}	---	.536	.536	.503	.536
G _{xz}	---	.664	.664	.780	.664
G _{xy}	.870	.870	.870	.870	.870
[0/90/+45/-45/-45/+45/90/0] Laminate Properties					
E _x	6.68	6.68	6.68	6.67	6.68
E _y	6.68	6.68	6.68	6.68	6.68
E _z	---	1.82	1.82	1.82	1.82
v _{yz}	---	.375	.376	.318	.375
v _{xz}	---	.375	.376	.317	.375
v _{xy}	.297	.297	.298	.296	.297
G _{yz}	---	.593	.593	.519	.593
G _{xz}	---	.593	.593	.627	.593
G _{xy}	2.58	2.57	2.57	2.57	2.57
[30/-30/-30/30] Laminate Properties					
E _x	6.84	6.84	6.84	6.84	6.84
E _y	1.77	1.77	1.77	1.77	1.77
E _z	---	1.66	1.66	1.50	1.66
v _{yz}	---	.390	.390	.434	.390
v _{xz}	---	-.095	-.094	-.197	-.095
v _{xy}	1.14	1.14	1.41	1.13	1.14
G _{yz}	---	.512	.512	.515	.512
G _{xz}	---	.705	.705	.708	.705
G _{xy}	3.43	3.43	3.42	3.42	3.43

Table 4

Correlation of Measured Composite Laminate Properties
with LAM3D Predictions
(E in units of MSI)

	IM7/PEEK		IM9/8551	
	Experimental (Gillespie 1994)	LAM3D (Chou et al. 1972)	Experimental (Gillespie 1994)	LAM3D (Chou et al. 1972)
E_x	11.98 ± 1.42	13.47	11.51 ± 1.21	13.70
E_y	2.53 ± 0.39	3.94	2.90 ± 0.22	3.97

Correlation of LAM3D failure analysis predictions with experimental measurements for the IM7/PEEK laminate is presented in Table 5. Excellent agreement is noted for the x-direction tension and y-direction compression load cases. Agreement in each case corresponds to ultimate longitudinal fiber direction failure (second ply failure in each load situation). It is noted that while the x-direction compression LAM3D prediction is overly conservative (e.g., 71.18 ksi predicted versus 99.20 ksi measured), the y-direction tension prediction is exceedingly high (e.g., 65.29 ksi predicted versus 34.53 ksi measured). In all cases, it is seen that the first ply failure prediction offers a very conservative laminate failure strength estimate.

Correlation of LAM3D failure analysis predictions with experimental measurements for the IM9/8551 laminate is presented in Table 6. Reasonable agreement is noted for the x-direction tension and x-direction compression load cases. As with the IM7/PEEK material, agreement in each case corresponds to ultimate longitudinal fiber direction failure. LAM3D laminate strength predictions are exceedingly high for the y-direction tension load case (e.g., 73.63 ksi predicted versus 42.39 ksi measured) as well as the y-direction compression load case (e.g., 66.57 ksi predicted versus 39.92 ksi measured). It is noted that for the IM9/8551 material, a first ply failure prediction offers overly conservative laminate strength predictions in three of the four load cases.

The discrepancies between the predicted and measured results are not surprising. It is a well-accepted fact that the prediction of ultimate laminate strength is not always done with great accuracy. This is largely because the direct translation of lamina properties into the laminate configuration is difficult to attain. Laminate strength predictions, much more so than the

mechanical property predictions, are subject to a wide degree of scatter. Laminate strengths can be influenced by many factors such as test specimen fabrication/preparation and specimen alignment in the load fixture as well as the type of test method selected. Furthermore, the particular lamina failure criterion employed as well as the assumptions made in the progressive ply failure analysis could have a significant effect on the ultimate laminate strength predictions.

4. CONCLUSIONS

An analytical model for predicting the effective three-dimensional thermo-mechanical properties and ultimate strength of thick laminated composite media under applied mechanical and/or thermal loadings was presented. The theoretical basis for the model is based on the work of Chou, Carleone, and Hsu (1972). Various types of lamina constitutive models were incorporated in the model. Numerous lamina failure criteria and a laminate failure analysis methodology that employs a progressive ply failure strategy were also developed. Model predictions were validated with comparisons between results in the published literature and available experimental data.

A major result of this work is the codification of a three-dimensional laminated media model into a user-friendly computer program environment, entitled LAM3D, for implementation as an engineering design tool for thick laminated composite structures. Several illustrative examples are presented to demonstrate the utility of LAM3D for predicting the effective properties and ultimate strength of laminated composite media.

This work represents an important step toward the development of a computationally efficient engineering tool, which will significantly enhance the design and analysis capability of a wide range thick composite structures for future military and commercial applications.

Table 5

Correlation of Measured Composite Laminate Strengths with LAM3D Predictions for an
IM7/PEEK [0₂/+45/-45/-45/+45/0₂] Laminate (units of ksi)

X- Direction Tension				
Experimentally Measured Strength:		135.80±3.30 (Gillespie 1994)		
LAM3D Failure Analysis Predictions:				
<u>Failure Iteration</u>	<u>Safety Factor</u>	<u>Ply</u>	<u>Mode</u>	
1	64.30	±45	S12	
2	134.57	0	X1T	

X - Direction Compression				
Experimentally Measured Strength:		99.20±2.95 (Gillespie 1994)		
LAM3D Failure Analysis Predictions:				
<u>Failure Iteration</u>	<u>Safety Factor</u>	<u>Ply</u>	<u>Mode</u>	
1	64.30	±45	S12	
2	71.18	0	X1C	

Y - Direction Tension				
Experimentally Measured Strength:		34.53±8.91 (Gillespie 1994)		
LAM3D Failure Analysis Predictions:				
<u>Failure Iteration</u>	<u>Safety Factor</u>	<u>Ply</u>	<u>Mode</u>	
1	25.35	±45	S12	
2	27.32	0	X2T	
3	39.81	±45	X2T	
4	65.29	±45	X1T	

Y - Direction Compression				
Experimentally Measured Strength:		49.13±2.43 (Gillespie 1994)		
LAM3D Failure Analysis Predictions:				
<u>Failure Iteration</u>	<u>Safety Factor</u>	<u>Ply</u>	<u>Mode</u>	
1	25.35	±45	S12	
2	45.68	±45	X1C	

Table 6

Correlation of Measured Composite Laminate Strengths with LAM3D Predictions for an IM9/8551 [0₂/+45/-45/-45/+45/0₂] Laminate (units of ksi)

X- Direction Tension

Experimentally Measured Strength: 186.25±7.97 (Gillespie 1994)

LAM3D Failure Analysis Predictions:

<u>Failure Iteration</u>	<u>Safety Factor</u>	<u>Ply</u>	<u>Mode</u>
1	156.34	±45	S12
2	213.89	0	X1T

X - Direction Compression

Experimentally Measured Strength: 105.24±5.85 (Gillespie 1994)

LAM3D Failure Analysis Predictions:

<u>Failure Iteration</u>	<u>Safety Factor</u>	<u>Ply</u>	<u>Mode</u>
1	88.90	0	X1C

Y - Direction Tension

Experimentally Measured Strength: 42.39±0.92 (Gillespie 1994)

LAM3D Failure Analysis Predictions:

<u>Failure Iteration</u>	<u>Safety Factor</u>	<u>Ply</u>	<u>Mode</u>
1	33.21	0	X2T
2	24.79	±45	S12
3	36.89	±45	X2T
4	73.63	0	X1C

Y - Direction Compression

Experimentally Measured Strength: 39.92±0.72 (Gillespie 1994)

LAM3D Failure Analysis Predictions:

<u>Failure Iteration</u>	<u>Safety Factor</u>	<u>Ply</u>	<u>Mode</u>
1	66.57	±45	X1C

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APPENDIX A
MATERIAL DATA BASE

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MATERIAL DATA BASE

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1          !      REGIONS TO ANALYZE
1          !      NUMBER OF REGIONS IN THE DATABASE
3          !      NUMBER OF MATERIALS IN DATABASE

```

```

1.0,    2.0,    5.0,    0.0  !  REG_ID,  FAIL_CRT,  ITERR,  ITEXT
0.0,    0.0,    0.0,    0.0  !  BETA,    PHI,      SI,      ---
8.0,    1.0,    0.0,    0.0  !  NPLY,    REG_TYP,  ---,    ---
2.0,    0.005,  0.0,    -200.0 !  MAT_ID,  THICKNESS, ANGLE,  TEMP
2.0,    0.005,  0.0,    -200.0 !  MAT_ID,  THICKNESS, ANGLE,  TEMP
2.0,    0.005,  45.0,   -200.0 !  MAT_ID,  THICKNESS, ANGLE,  TEMP
2.0,    0.005,  -45.0,   -200.0 !  MAT_ID,  THICKNESS, ANGLE,  TEMP
2.0,    0.005,  -45.0,   -200.0 !  MAT_ID,  THICKNESS, ANGLE,  TEMP
2.0,    0.005,  45.0,   -200.0 !  MAT_ID,  THICKNESS, ANGLE,  TEMP
2.0,    0.005,  0.0,    -200.0 !  MAT_ID,  THICKNESS, ANGLE,  TEMP
2.0,    0.005,  0.0,    -200.0 !  MAT_ID,  THICKNESS, ANGLE,  TEMP

```

```

1.0,      1.0,      0.100E+00 !  MAT_ID, MAT_TYPE, MAT_DENS(A1)
10.000E+06, 10.000E+06, 10.000E+06 !  E1,      E2,      E3
0.280E+00, 0.280E+00, 0.280E+00 !  NU23,    NU13,    NU12
3.906E+06, 3.906E+06, 3.906E+06 !  G23,      G13,      G12
0.0,      0.0,      0.0      !  A1,      A2,      A3
80.000E+03, 0.0,      0.0      !  (1)VM1,  ---,      --- (VonMises)
0.0,      0.0,      0.0      !  ---,      ---,      ---
0.0,      0.0,      0.0      !  ---,      ---,      ---
0.0,      0.0,      0.0      !  ---,      ---,      ---
0.0,      0.0,      0.0      !  ---,      ---,      ---
0.0,      0.0,      0.0      !  (2)X1T,  X2T,      X3T (MaxStress)
0.0,      0.0,      0.0      !  X1C,      X2C,      X3C
0.0,      0.0,      0.0      !  X23,      X13,      X12
0.0,      0.0,      0.0      !  ---,      ---,      ---
0.0,      0.0,      0.0      !  ---,      ---,      ---
0.0,      0.0,      0.0      !  (3) Y1T,  Y2T,      Y3T (MaxStrain)
0.0,      0.0,      0.0      !  Y1C,      Y2C,      Y3C
0.0,      0.0,      0.0      !  Y23,      Y13,      Y12
0.0,      0.0,      0.0      !  ---,      ---,      ---
0.0,      0.0,      0.0      !  ---,      ---,      ---
0.0,      0.0,      0.0      !  (4) X1T,  X2T,      HPD (Hydro-Pressure)
0.0,      0.0,      0.0      !  X1C(0),  X2C(0),  S
0.0,      0.0,      0.0      !  ML1,      ML2,      LTP
0.0,      0.0,      0.0      !  MT1,      MT2,      TTP
0.0,      0.0,      0.0      !  MS1,      MS2,      STP

```

0.0,	0.0,	0.0	!	(5)X1T,	X1C,	--- (Tsai-Wu)
0.0,	0.0,	0.0	!	X2T,	X2C,	---
0.0,	0.0,	0.0	!	S12,	F12,	F23
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	(6)E1,	E2,	E3 (Christensen's)
0.0,	0.0,	0.0	!	NU23,	NU13,	NU12
0.0,	0.0,	0.0	!	G23,	G13,	G12
0.0,	0.0,	0.0	!	K,	ALPHA,	---
0.0,	0.0,	0.0	!	Y1T,	Y1C,	---
0.0,	0.0,	0.0	!	(7) A1,	A11,	--- (Feng's)
0.0,	0.0,	0.0	!	A2,	A4,	---
0.0,	0.0,	0.0	!	A5,	A55,	---
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	(8) X1T,	X2T,	X3T (ModifiedHashin)
0.0,	0.0,	0.0	!	X1C,	X2C,	X3C
0.0,	0.0,	0.0	!	X23,	X13,	X12
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---

2.0,	1.0,	1.501E-04	!	MAT_ID,	MAT_TYPE,	MAT_DENS (Graphite/8551- Epoxy)
2.230E+06,	1.220E+06,	1.220E+06	!	E1,	E2,	E3
0.450E+00,	0.330E+00,	0.330E+00	!	NU23,	NU13,	NU12
0.700E+06,	0.700E+06,	0.700E+06	!	G23,	G13,	G12
-5.000E-07,	1.550E-05,	1.550E-05	!	A1,	A2,	A3
0.0,	0.0,	0.0	!	(1)VM1,	---	--- (VonMises)
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
420.0E+03,	8.7E+03,	8.7E+03	!	(2) X1T,	X2T,	X3T (MaxStress)
175.0E+03,	30.0E+03,	30.0E+03	!	X1C,	X2C,	X3C
20.0E+03,	20.0E+03,	20.0E+03	!	X23,	X13,	X12
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
0.0188,	0.0071,	0.0071	!	(3) Y1T,	Y2T,	Y3T (MaxStrain)
0.0078,	0.0246,	0.0246	!	Y1C,	Y2C,	Y3C
0.0290,	0.0290,	0.0290	!	Y23,	Y13,	Y12
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
420.0E+03,	8.7E+03,	1.0	!	(4) X1T,	X2T,	HPD (Hydro-Pressure)
175.0E+03,	100.0E+03,	20.30E+03	!	X1C(0),	X2C(0),	S
2.0,	2.0,	0.0	!	ML1,	ML2,	LTP
1.0,	1.0,	0.0	!	MT1,	MT2,	TTP

1.0,	1.0,	0.0	!	MS1,	MS2,	STP
420.0E+03,	175.0E+03,	0.0	!	(5) X1T,	X1C,	--- (Tsai-Wu)
8.7E+03,	30.0E+03,	0.0	!	X2T,	X2C,	---
20.0E+03,	1.1916E-09,	-1.916E-09	!	S12,	F12,	F23
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
2.230E+06,	1.220E+06,	1.220E+06	!	(6) E1,	E2,	E3 (Christensen's)
0.450E+00,	0.330E+00,	0.330E+00	!	NU23,	NU13,	NU12
0.700E+06,	0.700E+06,	0.700E+06	!	G23,	G13,	G12
1.507E-02,	0.1028,	0.0	!	K,	ALPHA,	---
0.0188,	0.0078,	0.0	!	Y1T,	Y1C,	---
452.55,	101910.0,	0.0	!	(7) A1,	A11,	--- (Feng's)
1189.1,	1189.1,	0.0	!	A2,	A4,	---
-37.42,	4819.74,	0.0	!	A5,	A55,	---
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
420.0E+03,	8.7E+03,	8.7E+03	!	(8) X1T,	X2T,	X3T (ModifiedHashin)
175.0E+03,	30.0E+03,	30.0E+03	!	X1C,	X2C,	X3C
20.0E+03,	20.0E+03,	20.0E+03	!	X23,	X13,	X12
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---

3.0,	1.0,	1.889E-04	!	MAT_ID,	MAT_TYPE,	MAT_DENS(S2-Glass/3501-Epoxy)
7.150E+06,	2.130E+06,	2.130E+06	!	E1,	E2,	E3
0.499E+00,	0.306E+00,	0.296E+00	!	NU23,	NU13,	NU12
0.710E+06,	0.980E+06,	0.980E+06	!	G23,	G13,	G12
2.300E-06,	1.850E-05,	1.850E-05	!	A1,	A2,	A3
0.0,	0.0,	0.0	!	(1) VM1,	---	--- (VonMises)
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
243.0E+03,	7.0E+03,	7.0E+03	!	(2) X1T,	X2T,	X3T (MaxStress)
177.0E+03,	30.6E+03,	35.0E+03	!	X1C,	X2C,	X3C
15.7E+03,	17.0E+03,	15.7E+03	!	X23,	X13,	X12
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
0.0340,	0.0033,	0.0040	!	(3) Y1T,	Y2T,	Y3T (MaxStrain)
0.0248,	0.0144,	0.0164	!	Y1C,	Y2C,	Y3C
0.0221,	0.0174,	0.0160	!	Y23,	Y13,	Y12
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	(4) X1T,	X2T,	HPD (Hydro-Pressure)
0.0,	0.0,	0.0	!	X1C(0),	X2C(0),	S
0.0,	0.0,	0.0	!	ML1,	ML2,	LTP

0.0,	0.0,	0.0	!	MT1,	MT2,	TTP
0.0,	0.0,	0.0	!	MS1,	MS2,	STP
0.0,	0.0,	0.0	!	(5) X1T,	X1C,	--- (Tsai-Wu)
0.0,	0.0,	0.0	!	X2T,	X2C,	---
0.0,	0.0,	0.0	!	S12,	F12,	F23
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	(6) E1,	E2,	E3 (Christensen's)
0.0,	0.0,	0.0	!	NU23,	NU13,	NU12
0.0,	0.0,	0.0	!	G23,	G13,	G12
0.0,	0.0,	0.0	!	K,	ALPHA,	---
0.0,	0.0,	0.0	!	Y1T,	Y1C,	---
0.0,	0.0,	0.0	!	(7) A1,	A11,	--- (Feng's)
0.0,	0.0,	0.0	!	A2,	A4,	---
0.0,	0.0,	0.0	!	A5,	A55,	---
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	(8) X1T,	X2T,	X3T (ModifiedHashin)
0.0,	0.0,	0.0	!	X1C,	X2C,	X3C
0.0,	0.0,	0.0	!	X23,	X13,	X12
0.0,	0.0,	0.0	!	---	---	---
0.0,	0.0,	0.0	!	---	---	---

APPENDIX B
SAMPLE OUTPUT FILE

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SAMPLE OUTPUT FILE

FROM OUTOFIN.F

 * OUTPUT FROM LAM3D (VERSION 3.0) *

TOTAL NUMBER OF PLIES > 8

TYPE OF LAMINA CONSTITUTIVE MODEL SELECTED >
 (1) ORTHOTROPIC (9 I.E.C.)

TYPE OF LAMINATE CONSTITUTIVE MODEL SELECTED >
 (2) EXACT (REF. CHOU)

TYPE OF FAILURE CRITERIA >
 (2) MAXIMUM STRESS

NUMBER OF ITERATIONS FOR FAILURE >
 10

 3-D LAMINA PROPERTIES

PLY= 1		
E1	E2	E3
0.223E+08	0.122E+07	0.122E+07
G23	G13	G12
0.700E+06	0.700E+06	0.700E+06
NU23	NU13	NU23
0.450	0.330	0.330

PLY= 2		
E1	E2	E3
0.223E+08	0.122E+07	0.122E+07
G23	G13	G12
0.700E+06	0.700E+06	0.700E+06
NU23	NU13	NU23
0.450	0.330	0.330

PLY= 3		
E1	E2	E3
0.223E+08	0.122E+07	0.122E+07
G23	G13	G12
0.700E+06	0.700E+06	0.700E+06
NU23	NU13	NU23
0.450	0.330	0.330

PLY= 4		
E1	E2	E3
0.223E+08	0.122E+07	0.122E+07
G23	G13	G12
0.700E+06	0.700E+06	0.700E+06
NU23	NU13	NU23
0.450	0.330	0.330

PLY=	5		
E1	E2	E3	
0.223E+08	0.122E+07	0.122E+07	
G23	G13	G12	
0.700E+06	0.700E+06	0.700E+06	
NU23	NU13	NU23	
0.450	0.330	0.330	

PLY=	6		
E1	E2	E3	
0.223E+08	0.122E+07	0.122E+07	
G23	G13	G12	
0.700E+06	0.700E+06	0.700E+06	
NU23	NU13	NU23	
0.450	0.330	0.330	

PLY=	7		
E1	E2	E3	
0.223E+08	0.122E+07	0.122E+07	
G23	G13	G12	
0.700E+06	0.700E+06	0.700E+06	
NU23	NU13	NU23	
0.450	0.330	0.330	

PLY=	8		
E1	E2	E3	
0.223E+08	0.122E+07	0.122E+07	
G23	G13	G12	
0.700E+06	0.700E+06	0.700E+06	
NU23	NU13	NU23	
0.450	0.330	0.330	

 FAILURE PARAMETERS

PLY=	1		
FR-1,1	FR-1,2	FR-1,3	
0.420E+06	0.870E+04	0.870E+04	
FR-2,1	FR-2,2	FR-2,3	
0.175E+06	0.300E+05	0.300E+05	
FR-3,1	FR-3,2	FR-3,3	
0.200E+05	0.200E+05	0.200E+05	
FR-4,1	FR-4,2	FR-4,3	
0.000E+00	0.000E+00	0.000E+00	
FR-5,1	FR-5,2	FR-5,3	
0.000E+00	0.000E+00	0.000E+00	

PLY=	2		
FR-1,1	FR-1,2	FR-1,3	
0.420E+06	0.870E+04	0.870E+04	
FR-2,1	FR-2,2	FR-2,3	
0.175E+06	0.300E+05	0.300E+05	
FR-3,1	FR-3,2	FR-3,3	
0.200E+05	0.200E+05	0.200E+05	
FR-4,1	FR-4,2	FR-4,3	

0.000E+00 0.000E+00 0.000E+00
 FR-5,1 FR-5,2 FR-5,3
 0.000E+00 0.000E+00 0.000E+00

PLY= 3
 FR-1,1 FR-1,2 FR-1,3
 0.420E+06 0.870E+04 0.870E+04
 FR-2,1 FR-2,2 FR-2,3
 0.175E+06 0.300E+05 0.300E+05
 FR-3,1 FR-3,2 FR-3,3
 0.200E+05 0.200E+05 0.200E+05
 FR-4,1 FR-4,2 FR-4,3
 0.000E+00 0.000E+00 0.000E+00
 FR-5,1 FR-5,2 FR-5,3
 0.000E+00 0.000E+00 0.000E+00

PLY= 4
 FR-1,1 FR-1,2 FR-1,3
 0.420E+06 0.870E+04 0.870E+04
 FR-2,1 FR-2,2 FR-2,3
 0.175E+06 0.300E+05 0.300E+05
 FR-3,1 FR-3,2 FR-3,3
 0.200E+05 0.200E+05 0.200E+05
 FR-4,1 FR-4,2 FR-4,3
 0.000E+00 0.000E+00 0.000E+00
 FR-5,1 FR-5,2 FR-5,3
 0.000E+00 0.000E+00 0.000E+00

PLY= 5
 FR-1,1 FR-1,2 FR-1,3
 0.420E+06 0.870E+04 0.870E+04
 FR-2,1 FR-2,2 FR-2,3
 0.175E+06 0.300E+05 0.300E+05
 FR-3,1 FR-3,2 FR-3,3
 0.200E+05 0.200E+05 0.200E+05
 FR-4,1 FR-4,2 FR-4,3
 0.000E+00 0.000E+00 0.000E+00
 FR-5,1 FR-5,2 FR-5,3
 0.000E+00 0.000E+00 0.000E+00

PLY= 6
 FR-1,1 FR-1,2 FR-1,3
 0.420E+06 0.870E+04 0.870E+04
 FR-2,1 FR-2,2 FR-2,3
 0.175E+06 0.300E+05 0.300E+05
 FR-3,1 FR-3,2 FR-3,3
 0.200E+05 0.200E+05 0.200E+05
 FR-4,1 FR-4,2 FR-4,3
 0.000E+00 0.000E+00 0.000E+00
 FR-5,1 FR-5,2 FR-5,3
 0.000E+00 0.000E+00 0.000E+00

PLY= 7
 FR-1,1 FR-1,2 FR-1,3
 0.420E+06 0.870E+04 0.870E+04
 FR-2,1 FR-2,2 FR-2,3
 0.175E+06 0.300E+05 0.300E+05
 FR-3,1 FR-3,2 FR-3,3

0.200E+05 0.200E+05 0.200E+05
 FR-4,1 FR-4,2 FR-4,3
 0.000E+00 0.000E+00 0.000E+00
 FR-5,1 FR-5,2 FR-5,3
 0.000E+00 0.000E+00 0.000E+00

PLY= 8
 FR-1,1 FR-1,2 FR-1,3
 0.420E+06 0.870E+04 0.870E+04
 FR-2,1 FR-2,2 FR-2,3
 0.175E+06 0.300E+05 0.300E+05
 FR-3,1 FR-3,2 FR-3,3
 0.200E+05 0.200E+05 0.200E+05
 FR-4,1 FR-4,2 FR-4,3
 0.000E+00 0.000E+00 0.000E+00
 FR-5,1 FR-5,2 FR-5,3
 0.000E+00 0.000E+00 0.000E+00

 LAMINATE GEOMETRY

PLY	ANGLE (DEGREES)	THICKNESS (IN)	TEMP (F)
1	0.0000E+00	0.5000E-02	-200.0
2	0.0000E+00	0.5000E-02	-200.0
3	45.00	0.5000E-02	-200.0
4	-45.00	0.5000E-02	-200.0
5	-45.00	0.5000E-02	-200.0
6	45.00	0.5000E-02	-200.0
7	0.0000E+00	0.5000E-02	-200.0
8	0.0000E+00	0.5000E-02	-200.0

 LAMINATE ORIENTATION WITHIN GLOBAL SYSTEM

BETA = 0.0000000E+00
 PHI = 0.0000000E+00
 SI = 0.0000000E+00

 APPLIED GLOBAL MECHANICAL STRESSES

X = -1000.000
 Y = 0.0000000E+00
 Z = 0.0000000E+00
 YZ = 10.00000
 XZ = 35.00000
 XY = 10.00000

 APPLIED LOCAL MECHANICAL STRESSES

X = -1000.000
 Y = 0.0000000E+00
 Z = 0.0000000E+00
 YZ = 10.00000
 XZ = 35.00000
 XY = 10.00000

0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.7000E+06

ALPHA GLOBAL MATRIX >

PLY: 1
-0.5000E-06 0.1550E-04 0.1550E-04 0.0000E+00 0.0000E+00 0.0000E+00
PLY: 2
-0.5000E-06 0.1550E-04 0.1550E-04 0.0000E+00 0.0000E+00 0.0000E+00
PLY: 3
0.7500E-05 0.7500E-05 0.1550E-04 0.0000E+00 0.0000E+00 -0.1600E-04
PLY: 4
0.7500E-05 0.7500E-05 0.1550E-04 0.0000E+00 0.0000E+00 0.1600E-04
PLY: 5
0.7500E-05 0.7500E-05 0.1550E-04 0.0000E+00 0.0000E+00 0.1600E-04
PLY: 6
0.7500E-05 0.7500E-05 0.1550E-04 0.0000E+00 0.0000E+00 -0.1600E-04
PLY: 7
-0.5000E-06 0.1550E-04 0.1550E-04 0.0000E+00 0.0000E+00 0.0000E+00
PLY: 8
-0.5000E-06 0.1550E-04 0.1550E-04 0.0000E+00 0.0000E+00 0.0000E+00

CM GLOBAL STIFFNESS MATRIX >

0.1498E+08 0.3255E+07 0.7394E+06 0.0000E+00 0.0000E+00 0.0000E+00
0.3255E+07 0.4358E+07 0.7218E+06 0.0000E+00 0.0000E+00 0.0000E+00
0.7394E+06 0.7218E+06 0.1554E+07 0.0000E+00 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.7000E+06 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.7000E+06 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.3206E+07

FROM OUTEFPR.F

EFFECTIVE LAMINATE PROPERTIES >

EX = 1.2518537E+07
EY = 3442994.
EZ = 1431594.
NUYZ = 0.3696677
NUXZ = 0.1396095
NUXY = 0.7237686
NUZY = 0.1537075
NUZX = 1.5965447E-02
NUYX = 0.1990592
GYZ = 699999.9
GXZ = 699999.9
GXY = 3206368.
ALPHAX = -6.6476514E-07
ALPHAY = 3.6833085E-06
ALPHAZ = 2.0974869E-05
ALPHAYZ = 0.0000000E+00
ALPHAXZ = 0.0000000E+00
ALPHAXY = -2.9743138E-13

FROM OUTLOAD.F

AVERAGE MECHANICAL STRESSES (PSI.) >

X = -1000.000
Y = 0.0000000E+00
Z = 0.0000000E+00
YZ = 10.00000
XZ = 35.00000
XY = 10.00000

AVERAGE THERMAL STRESSES (PSI.) >

X-TH = -3508.091
Y-TH = -2.0268120E-04
Z-TH = 2.0282280E-04
YZ-TH = 3.9123173E-05
XZ-TH = 5.5890254E-11
XY-TH = 7.9843231E-17

MECHANICAL LAMINATE STRAINS >

EPSX = -7.9881538E-05
EPSY = 5.7815741E-05
EPSZ = 1.1152222E-05
EPSYZ = 1.4285716E-05
EPSXZ = 5.0000006E-05
EPSXY = 3.1187940E-06

THERMAL LAMINATE STRAINS >

EPSX-TH = -2.0268120E-04
EPSY-TH = 2.0282280E-04
EPSZ-TH = 3.9123173E-05
EPSYZ-TH = 5.5890254E-11
EPSXZ-TH = 7.9843231E-17
EPSXY-TH = 2.4901459E-23

FROM OUTMESTR.F

MECHANICAL GLOBAL PLY STRAIN

PLY	EPS-X	EPS-Y	EPS-Z	EPS-YZ	EPS-XZ	EPS-XY
1	-.799E-04	0.578E-04	0.119E-04	0.143E-04	0.500E-04	0.312E-05
2	-.799E-04	0.578E-04	0.119E-04	0.143E-04	0.500E-04	0.312E-05
3	-.799E-04	0.578E-04	0.103E-04	0.143E-04	0.500E-04	0.312E-05
4	-.799E-04	0.578E-04	0.104E-04	0.143E-04	0.500E-04	0.312E-05
5	-.799E-04	0.578E-04	0.104E-04	0.143E-04	0.500E-04	0.312E-05
6	-.799E-04	0.578E-04	0.103E-04	0.143E-04	0.500E-04	0.312E-05
7	-.799E-04	0.578E-04	0.119E-04	0.143E-04	0.500E-04	0.312E-05
8	-.799E-04	0.578E-04	0.119E-04	0.143E-04	0.500E-04	0.312E-05

MECHANICAL GLOBAL PLY STRESS

PLY	SIG-X	SIG-Y	SIG-Z	SIG-YZ	SIG-XZ	SIG-XY
1	-.177E+04	38.6	0.000E+00	10.0	35.0	2.18
2	-.177E+04	38.6	0.000E+00	10.0	35.0	2.18
3	-215.	-22.1	0.000E+00	10.0	35.0	-99.2
4	-248.	-55.1	0.000E+00	10.0	35.0	135.
5	-248.	-55.1	0.000E+00	10.0	35.0	135.
6	-215.	-22.1	0.000E+00	10.0	35.0	-99.2
7	-.177E+04	38.6	0.000E+00	10.0	35.0	2.18
8	-.177E+04	38.6	0.000E+00	10.0	35.0	2.18

MECHANICAL PRINCIPAL PLY STRAIN

PLY	EPS-1	EPS-2	EPS-3	EPS-23	EPS-13	EPS-12
1	-.799E-04	0.578E-04	0.119E-04	0.143E-04	0.500E-04	0.312E-05
2	-.799E-04	0.578E-04	0.119E-04	0.143E-04	0.500E-04	0.312E-05
3	-.947E-05	-.126E-04	0.103E-04	-.253E-04	0.455E-04	0.138E-03
4	-.126E-04	-.947E-05	0.104E-04	0.455E-04	0.253E-04	-.138E-03
5	-.126E-04	-.947E-05	0.104E-04	0.455E-04	0.253E-04	-.138E-03
6	-.947E-05	-.126E-04	0.103E-04	-.253E-04	0.455E-04	0.138E-03
7	-.799E-04	0.578E-04	0.119E-04	0.143E-04	0.500E-04	0.312E-05
8	-.799E-04	0.578E-04	0.119E-04	0.143E-04	0.500E-04	0.312E-05

MECHANICAL PRINCIPAL PLY STRESS

PLY	SIG-1	SIG-2	SIG-3	SIG-23	SIG-13	SIG-12
1	-.177E+04	38.6	0.000E+00	10.0	35.0	2.18
2	-.177E+04	38.6	0.000E+00	10.0	35.0	2.18
3	-.218.	-19.3	0.000E+00	-17.7	31.8	96.4
4	-.286.	-16.7	0.000E+00	31.8	17.7	-96.4
5	-.286.	-16.7	0.000E+00	31.8	17.7	-96.4
6	-.218.	-19.3	0.000E+00	-17.7	31.8	96.4
7	-.177E+04	38.6	0.000E+00	10.0	35.0	2.18
8	-.177E+04	38.6	0.000E+00	10.0	35.0	2.18

FROM OUTTHSTR.F

THERMAL GLOBAL PLY STRAIN

PLY	EPS-X	EPS-Y	EPS-Z	EPS-YZ	EPS-XZ	EPS-XY
1	-.203E-03	0.203E-03	0.453E-05	0.559E-10	0.798E-16	0.249E-22
2	-.203E-03	0.203E-03	0.453E-05	0.559E-10	0.798E-16	0.249E-22
3	-.203E-03	0.203E-03	-.664E-07	0.559E-10	0.798E-16	0.249E-22
4	-.203E-03	0.203E-03	-.664E-07	0.559E-10	0.798E-16	0.249E-22
5	-.203E-03	0.203E-03	-.664E-07	0.559E-10	0.798E-16	0.249E-22
6	-.203E-03	0.203E-03	-.664E-07	0.559E-10	0.798E-16	0.249E-22
7	-.203E-03	0.203E-03	0.453E-05	0.559E-10	0.798E-16	0.249E-22
8	-.203E-03	0.203E-03	0.453E-05	0.559E-10	0.798E-16	0.249E-22

THERMAL GLOBAL PLY STRESS

PLY	SIG-X	SIG-Y	SIG-Z	SIG-YZ	SIG-XZ	SIG-XY
1	-.211E+04	0.712E+04	0.203E-03	0.391E-04	0.559E-10	0.174E-16
2	-.211E+04	0.712E+04	0.203E-03	0.391E-04	0.559E-10	0.174E-16
3	0.437E+04	0.494E+04	0.203E-03	0.391E-04	0.559E-10	-.230E+04
4	0.437E+04	0.494E+04	0.203E-03	0.391E-04	0.559E-10	0.230E+04
5	0.437E+04	0.494E+04	0.203E-03	0.391E-04	0.559E-10	0.230E+04
6	0.437E+04	0.494E+04	0.203E-03	0.391E-04	0.559E-10	-.230E+04
7	-.211E+04	0.712E+04	0.203E-03	0.391E-04	0.559E-10	0.174E-16
8	-.211E+04	0.712E+04	0.203E-03	0.391E-04	0.559E-10	0.174E-16

THERMAL PRINCIPAL PLY STRAIN

PLY	EPS-1	EPS-2	EPS-3	EPS-23	EPS-13	EPS-12
1	-.203E-03	0.203E-03	0.453E-05	0.559E-10	0.798E-16	0.249E-22
2	-.203E-03	0.203E-03	0.453E-05	0.559E-10	0.798E-16	0.249E-22
3	0.708E-07	0.708E-07	-.664E-07	0.395E-10	0.395E-10	0.406E-03
4	0.708E-07	0.708E-07	-.664E-07	0.395E-10	-.395E-10	-.406E-03
5	0.708E-07	0.708E-07	-.664E-07	0.395E-10	-.395E-10	-.406E-03
6	0.708E-07	0.708E-07	-.664E-07	0.395E-10	0.395E-10	0.406E-03
7	-.203E-03	0.203E-03	0.453E-05	0.559E-10	0.798E-16	0.249E-22
8	-.203E-03	0.203E-03	0.453E-05	0.559E-10	0.798E-16	0.249E-22

THERMAL PRINCIPAL PLY STRESS

PLY	SIG-1	SIG-2	SIG-3	SIG-23	SIG-13	SIG-12
1	-.211E+04	0.712E+04	0.203E-03	0.391E-04	0.559E-10	0.174E-16
2	-.211E+04	0.712E+04	0.203E-03	0.391E-04	0.559E-10	0.174E-16
3	0.236E+04	0.695E+04	0.203E-03	0.277E-04	0.277E-04	284.
4	0.236E+04	0.695E+04	0.203E-03	0.277E-04	-.277E-04	-284.
5	0.236E+04	0.695E+04	0.203E-03	0.277E-04	-.277E-04	-284.
6	0.236E+04	0.695E+04	0.203E-03	0.277E-04	0.277E-04	284.
7	-.211E+04	0.712E+04	0.203E-03	0.391E-04	0.559E-10	0.174E-16
8	-.211E+04	0.712E+04	0.203E-03	0.391E-04	0.559E-10	0.174E-16

FROM OUTFAIL.F

ITERATION NUMBER >

1

FAILURE ASSESSMENT...

PLY > 1 MODE > X2T
 PLY > 2 MODE > X2T
 PLY > 7 MODE > X2T
 PLY > 8 MODE > X2T
 SAFTY FACTOR > 40.91054

FROM OUTCIJS.F.....

CIJ PRINCIPAL MATRIX >

PLY: 1

0.2230E+08	-0.7333	-2.363	0.0000E+00	0.0000E+00	0.0000E+00
-0.7333	-0.4024E-05	-2.222	0.0000E+00	0.0000E+00	0.0000E+00
-2.363	-2.222	-4.938	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

PLY: 2

0.2230E+08	-0.7333	-2.363	0.0000E+00	0.0000E+00	0.0000E+00
-0.7333	-0.4024E-05	-2.222	0.0000E+00	0.0000E+00	0.0000E+00
-2.363	-2.222	-4.938	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

PLY: 3

0.2279E+08	0.7482E+06	0.7482E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.1554E+07	0.7130E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.7130E+06	0.1554E+07	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

PLY: 4

0.2279E+08	0.7482E+06	0.7482E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.1554E+07	0.7130E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.7130E+06	0.1554E+07	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

PLY: 5

0.2279E+08	0.7482E+06	0.7482E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.1554E+07	0.7130E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.7130E+06	0.1554E+07	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

PLY: 6

0.2279E+08	0.7482E+06	0.7482E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.1554E+07	0.7130E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.7130E+06	0.1554E+07	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

PLY: 7

0.2230E+08	-0.7333	-2.363	0.0000E+00	0.0000E+00	0.0000E+00
-0.7333	-0.4024E-05	-2.222	0.0000E+00	0.0000E+00	0.0000E+00
-2.363	-2.222	-4.938	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

PLY: 8
 0.2230E+08 -0.7333 -2.363 0.0000E+00 0.0000E+00 0.0000E+00
 -0.7333 -0.4024E-05 -2.222 0.0000E+00 0.0000E+00 0.0000E+00
 -2.363 -2.222 -4.938 0.0000E+00 0.0000E+00 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.7000E+06 0.0000E+00 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.7000E+06 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.7000E+06

ALPHA GLOBAL MATRIX >

PLY: 1
 -0.5000E-06 0.1550E-04 0.1550E-04 0.0000E+00 0.0000E+00 0.0000E+00
 PLY: 2
 -0.5000E-06 0.1550E-04 0.1550E-04 0.0000E+00 0.0000E+00 0.0000E+00
 PLY: 3
 0.7500E-05 0.7500E-05 0.1550E-04 0.0000E+00 0.0000E+00 -0.1600E-04
 PLY: 4
 0.7500E-05 0.7500E-05 0.1550E-04 0.0000E+00 0.0000E+00 0.1600E-04
 PLY: 5
 0.7500E-05 0.7500E-05 0.1550E-04 0.0000E+00 0.0000E+00 0.1600E-04
 PLY: 6
 0.7500E-05 0.7500E-05 0.1550E-04 0.0000E+00 0.0000E+00 -0.1600E-04
 PLY: 7
 -0.5000E-06 0.1550E-04 0.1550E-04 0.0000E+00 0.0000E+00 0.0000E+00
 PLY: 8
 -0.5000E-06 0.1550E-04 0.1550E-04 0.0000E+00 0.0000E+00 0.0000E+00

CM GLOBAL STIFFNESS MATRIX >

0.1456E+08 0.2709E+07 -4.684 0.0000E+00 0.0000E+00 0.0000E+00
 0.2709E+07 0.3409E+07 -4.543 0.0000E+00 0.0000E+00 0.0000E+00
 -4.684 -4.543 -9.814 0.0000E+00 0.0000E+00 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.7000E+06 0.0000E+00 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.7000E+06 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.3206E+07

FROM OUTEFPR.F

EFFECTIVE LAMINATE PROPERTIES >

EX = 1.2406259E+07
 EY = 2904853.
 EZ = -9.814123
 NUYZ = 0.3741388
 NUXZ = 0.1094047
 NUXY = 0.7946534
 NUZY = -1.2640380E-06
 NUZX = -8.6545938E-08
 NUYX = 0.1860634
 GYZ = 699999.9
 GXZ = 699999.9
 GXY = 3206367.
 ALPHAX = -4.0998452E-07
 ALPHAY = 1.3799854E-06
 ALPHAZ = -1.771376
 ALPHAYZ = 0.0000000E+00
 ALPHAXZ = 0.0000000E+00
 ALPHAXY = -2.9743140E-13

FROM OUTLOAD.F

AVERAGE MECHANICAL STRESSES (PSI.) >

X = -1000.000

Y = 0.000000E+00
 Z = 0.000000E+00
 YZ = 10.00000
 XZ = 35.00000
 XY = 10.00000

AVERAGE THERMAL STRESSES (PSI.) >

X-TH = -1213.327
 Y-TH = -6.7138564E-05
 Z-TH = 7.7716759E-05
 YZ-TH = 2.7808765E-06
 XZ-TH = 3.9726811E-12
 XY-TH = 5.6752597E-18

MECHANICAL LAMINATE STRAINS >

EPSX = -8.0604470E-05
 EPSY = 6.4052612E-05
 EPSZ = 8.8185079E-06
 EPSYZ = 1.4285716E-05
 EPSXZ = 5.0000006E-05
 EPSXY = 3.1187942E-06

THERMAL LAMINATE STRAINS >

EPSX-TH = -6.7138564E-05
 EPSY-TH = 7.7716759E-05
 EPSZ-TH = 2.7808765E-06
 EPSYZ-TH = 3.9726811E-12
 EPSXZ-TH = 5.6752597E-18
 EPSXY-TH = 1.7699968E-24

FROM OUTMESTR.F

MECHANICAL GLOBAL PLY STRAIN

PLY	EPS-X	EPS-Y	EPS-Z	EPS-YZ	EPS-XZ	EPS-XY
1	-.806E-04	0.641E-04	0.975E-05	0.143E-04	0.500E-04	0.312E-05
2	-.806E-04	0.641E-04	0.975E-05	0.143E-04	0.500E-04	0.312E-05
3	-.806E-04	0.641E-04	0.774E-05	0.143E-04	0.500E-04	0.312E-05
4	-.806E-04	0.641E-04	0.782E-05	0.143E-04	0.500E-04	0.312E-05
5	-.806E-04	0.641E-04	0.782E-05	0.143E-04	0.500E-04	0.312E-05
6	-.806E-04	0.641E-04	0.774E-05	0.143E-04	0.500E-04	0.312E-05
7	-.806E-04	0.641E-04	0.975E-05	0.143E-04	0.500E-04	0.312E-05
8	-.806E-04	0.641E-04	0.975E-05	0.143E-04	0.500E-04	0.312E-05

MECHANICAL GLOBAL PLY STRESS

PLY	SIG-X	SIG-Y	SIG-Z	SIG-YZ	SIG-XZ	SIG-XY
1	-.180E+04	0.375E-04	0.000E+00	10.0	35.0	2.18
2	-.180E+04	0.375E-04	0.000E+00	10.0	35.0	2.18
3	-186.	16.5	0.000E+00	10.0	35.0	-69.9
4	-219.	-16.5	0.000E+00	10.0	35.0	106.
5	-219.	-16.5	0.000E+00	10.0	35.0	106.
6	-186.	16.5	0.000E+00	10.0	35.0	-69.9
7	-.180E+04	0.375E-04	0.000E+00	10.0	35.0	2.18
8	-.180E+04	0.375E-04	0.000E+00	10.0	35.0	2.18

MECHANICAL PRINCIPAL PLY STRAIN

PLY	EPS-1	EPS-2	EPS-3	EPS-23	EPS-13	EPS-12
1	-.806E-04	0.641E-04	0.975E-05	0.143E-04	0.500E-04	0.312E-05
2	-.806E-04	0.641E-04	0.975E-05	0.143E-04	0.500E-04	0.312E-05
3	-.672E-05	-.984E-05	0.774E-05	-.253E-04	0.455E-04	0.145E-03

4	-.984E-05	-.672E-05	0.782E-05	0.455E-04	0.253E-04	-.145E-03
5	-.984E-05	-.672E-05	0.782E-05	0.455E-04	0.253E-04	-.145E-03
6	-.672E-05	-.984E-05	0.774E-05	-.253E-04	0.455E-04	0.145E-03
7	-.806E-04	0.641E-04	0.975E-05	0.143E-04	0.500E-04	0.312E-05
8	-.806E-04	0.641E-04	0.975E-05	0.143E-04	0.500E-04	0.312E-05

MECHANICAL PRINCIPAL PLY STRESS

PLY	SIG-1	SIG-2	SIG-3	SIG-23	SIG-13	SIG-12
1	-.180E+04	0.375E-04	0.000E+00	10.0	35.0	2.18
2	-.180E+04	0.375E-04	0.000E+00	10.0	35.0	2.18
3	-155.	-14.8	0.000E+00	-17.7	31.8	101.
4	-223.	-12.2	0.000E+00	31.8	17.7	-101.
5	-223.	-12.2	0.000E+00	31.8	17.7	-101.
6	-155.	-14.8	0.000E+00	-17.7	31.8	101.
7	-.180E+04	0.375E-04	0.000E+00	10.0	35.0	2.18
8	-.180E+04	0.375E-04	0.000E+00	10.0	35.0	2.18

FROM OUTTHSTR.F

THERMAL GLOBAL PLY STRAIN

PLY	EPS-X	EPS-Y	EPS-Z	EPS-YZ	EPS-XZ	EPS-XY
1	-.671E-04	0.777E-04	-.186E-04	0.397E-11	0.568E-17	0.177E-23
2	-.671E-04	0.777E-04	-.186E-04	0.397E-11	0.568E-17	0.177E-23
3	-.671E-04	0.777E-04	-.497E-05	0.397E-11	0.568E-17	0.177E-23
4	-.671E-04	0.777E-04	-.497E-05	0.397E-11	0.568E-17	0.177E-23
5	-.671E-04	0.777E-04	-.497E-05	0.397E-11	0.568E-17	0.177E-23
6	-.671E-04	0.777E-04	-.497E-05	0.397E-11	0.568E-17	0.177E-23
7	-.671E-04	0.777E-04	-.186E-04	0.397E-11	0.568E-17	0.177E-23
8	-.671E-04	0.777E-04	-.186E-04	0.397E-11	0.568E-17	0.177E-23

THERMAL GLOBAL PLY STRESS

PLY	SIG-X	SIG-Y	SIG-Z	SIG-YZ	SIG-XZ	SIG-XY
1	-.373E+04	-.673E-02	0.777E-04	0.278E-05	0.397E-11	0.124E-17
2	-.373E+04	-.673E-02	0.777E-04	0.278E-05	0.397E-11	0.124E-17
3	0.462E+04	0.482E-04	0.777E-04	0.278E-05	0.397E-11	-.224E+04
4	0.462E+04	0.482E+04	0.777E-04	0.278E-05	0.397E-11	0.224E+04
5	0.462E+04	0.482E+04	0.777E-04	0.278E-05	0.397E-11	0.224E+04
6	0.462E+04	0.482E+04	0.777E-04	0.278E-05	0.397E-11	-.224E+04
7	-.373E+04	-.673E-02	0.777E-04	0.278E-05	0.397E-11	0.124E-17
8	-.373E+04	-.673E-02	0.777E-04	0.278E-05	0.397E-11	0.124E-17

THERMAL PRINCIPAL PLY STRAIN

PLY	EPS-1	EPS-2	EPS-3	EPS-23	EPS-13	EPS-12
1	-.671E-04	0.777E-04	-.186E-04	0.397E-11	0.568E-17	0.177E-23
2	-.671E-04	0.777E-04	-.186E-04	0.397E-11	0.568E-17	0.177E-23
3	0.529E-05	0.529E-05	-.497E-05	0.281E-11	0.281E-11	0.145E-03
4	0.529E-05	0.529E-05	-.497E-05	0.281E-11	-.281E-11	-.145E-03
5	0.529E-05	0.529E-05	-.497E-05	0.281E-11	-.281E-11	-.145E-03
6	0.529E-05	0.529E-05	-.497E-05	0.281E-11	0.281E-11	0.145E-03
7	-.671E-04	0.777E-04	-.186E-04	0.397E-11	0.568E-17	0.177E-23
8	-.671E-04	0.777E-04	-.186E-04	0.397E-11	0.568E-17	0.177E-23

THERMAL PRINCIPAL PLY STRESS

PLY	SIG-1	SIG-2	SIG-3	SIG-23	SIG-13	SIG-12
1	-.373E+04	-.673E-02	0.777E-04	0.278E-05	0.397E-11	0.124E-17
2	-.373E+04	-.673E-02	0.777E-04	0.278E-05	0.397E-11	0.124E-17
3	0.248E+04	0.696E+04	0.777E-04	0.197E-05	0.197E-05	101.
4	0.248E+04	0.696E+04	0.777E-04	0.197E-05	-.197E-05	-101.
5	0.248E+04	0.696E+04	0.777E-04	0.197E-05	-.197E-05	-101.

6 0.248E+04 0.696E+04 0.777E-04 0.197E-05 0.197E-05 101.
 7 -0.373E+04 -0.673E-02 0.777E-04 0.278E-05 0.397E-11 0.124E-17
 8 -0.373E+04 -0.673E-02 0.777E-04 0.278E-05 0.397E-11 0.124E-17

FROM OUTFAIL.F
 ITERATION NUMBER > 2
 FAILURE ASSESSMENT...
 PLY > 1 MODE > X1C
 PLY > 1 MODE > X2T
 PLY > 2 MODE > X1C
 PLY > 2 MODE > X2T
 PLY > 7 MODE > X1C
 PLY > 7 MODE > X2T
 PLY > 8 MODE > X1C
 PLY > 8 MODE > X2T
 SAFTY FACTOR > 95.28498

FROM OUTCIJS.F.....
 CIJ PRINCIPAL MATRIX >

PLY: 1

0.4946	-0.3627	-1.169	0.0000E+00	0.0000E+00	0.0000E+00
-0.3627	0.2660	-1.365	0.0000E+00	0.0000E+00	0.0000E+00
-1.169	-1.365	-2.177	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

 PLY: 2

0.4946	-0.3627	-1.169	0.0000E+00	0.0000E+00	0.0000E+00
-0.3627	0.2660	-1.365	0.0000E+00	0.0000E+00	0.0000E+00
-1.169	-1.365	-2.177	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

 PLY: 3

0.2279E+08	0.7482E+06	0.7482E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.1554E+07	0.7130E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.7130E+06	0.1554E+07	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

 PLY: 4

0.2279E+08	0.7482E+06	0.7482E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.1554E+07	0.7130E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.7130E+06	0.1554E+07	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

 PLY: 5

0.2279E+08	0.7482E+06	0.7482E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.1554E+07	0.7130E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.7130E+06	0.1554E+07	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

 PLY: 6

0.2279E+08	0.7482E+06	0.7482E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.1554E+07	0.7130E+06	0.0000E+00	0.0000E+00	0.0000E+00
0.7482E+06	0.7130E+06	0.1554E+07	0.0000E+00	0.0000E+00	0.0000E+00

0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.5310E+07	0.5310E+07	0.1762E+05	0.0000E+00	0.0000E+00	0.5713E+07

PLY: 7

0.4946	-0.3627	-1.169	0.0000E+00	0.0000E+00	0.0000E+00
-0.3627	0.2660	-1.365	0.0000E+00	0.0000E+00	0.0000E+00
-1.169	-1.365	-2.177	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

PLY: 8

0.4946	-0.3627	-1.169	0.0000E+00	0.0000E+00	0.0000E+00
-0.3627	0.2660	-1.365	0.0000E+00	0.0000E+00	0.0000E+00
-1.169	-1.365	-2.177	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06

ALPHA GLOBAL MATRIX >

PLY: 1

-0.5000E-06	0.1550E-04	0.1550E-04	0.0000E+00	0.0000E+00	0.0000E+00
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PLY: 2

-0.5000E-06	0.1550E-04	0.1550E-04	0.0000E+00	0.0000E+00	0.0000E+00
-------------	------------	------------	------------	------------	------------

PLY: 3

0.7500E-05	0.7500E-05	0.1550E-04	0.0000E+00	0.0000E+00	-0.1600E-04
------------	------------	------------	------------	------------	-------------

PLY: 4

0.7500E-05	0.7500E-05	0.1550E-04	0.0000E+00	0.0000E+00	0.1600E-04
------------	------------	------------	------------	------------	------------

PLY: 5

0.7500E-05	0.7500E-05	0.1550E-04	0.0000E+00	0.0000E+00	0.1600E-04
------------	------------	------------	------------	------------	------------

PLY: 6

0.7500E-05	0.7500E-05	0.1550E-04	0.0000E+00	0.0000E+00	-0.1600E-04
------------	------------	------------	------------	------------	-------------

PLY: 7

-0.5000E-06	0.1550E-04	0.1550E-04	0.0000E+00	0.0000E+00	0.0000E+00
-------------	------------	------------	------------	------------	------------

PLY: 8

-0.5000E-06	0.1550E-04	0.1550E-04	0.0000E+00	0.0000E+00	0.0000E+00
-------------	------------	------------	------------	------------	------------

CM GLOBAL STIFFNESS MATRIX >

0.3409E+07	0.2709E+07	-2.192	0.0000E+00	0.0000E+00	0.0000E+00
0.2709E+07	0.3409E+07	-2.388	0.0000E+00	0.0000E+00	0.0000E+00
-2.192	-2.388	-4.291	0.0000E+00	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.7000E+06	0.0000E+00
0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.3206E+07

FROM OUTEFPR.F

EFFECTIVE LAMINATE PROPERTIES >

EX = 1256259.
 EY = 1256259.
 EZ = -4.290609
 NUYZ = 0.1506686
 NUXZ = 6.8527453E-02
 NUXY = 0.7946534
 NUZY = -5.1459142E-07
 NUZX = -2.3404780E-07
 NUYX = 0.7946537
 GYZ = 699999.8
 GXZ = 699999.8
 GXY = 3206367.

ALPHAX = 9.5462826E-07
 ALPHAY = -1.8207174E-07
 ALPHAZ = -4.051761
 ALPHAYZ = 0.0000000E+00
 ALPHAXZ = 0.0000000E+00
 ALPHAXY = -2.9743140E-13

FROM OUTLOAD.F

AVERAGE MECHANICAL STRESSES (PSI.) >

X = -1000.000
 Y = 0.0000000E+00
 Z = 0.0000000E+00
 YZ = 10.00000
 XZ = 35.00000
 XY = 10.00000

AVERAGE THERMAL STRESSES (PSI.) >

X-TH = -2328.330
 Y-TH = -1.6637220E-03
 Z-TH = 1.4727962E-03
 YZ-TH = -2.1625223E-04
 XZ-TH = -3.0893182E-10
 XY-TH = -4.4133127E-16

MECHANICAL LAMINATE STRAINS >

EPSX = -7.9601456E-04
 EPSY = 6.3255552E-04
 EPSZ = 5.4548957E-05
 EPSYZ = 1.4285717E-05
 EPSXZ = 5.0000010E-05
 EPSXY = 3.1187942E-06

THERMAL LAMINATE STRAINS >

EPSX-TH = -1.6637220E-03
 EPSY-TH = 1.4727962E-03
 EPSZ-TH = -2.1625223E-04
 EPSYZ-TH = -3.0893182E-10
 EPSXZ-TH = -4.4133127E-16
 EPSXY-TH = -1.3764214E-22

FROM OUTMESTR.F

MECHANICAL GLOBAL PLY STRAIN

PLY	EPS-X	EPS-Y	EPS-Z	EPS-YZ	EPS-XZ	EPS-XY
1	-.796E-03	0.633E-03	0.307E-04	0.143E-04	0.500E-04	0.312E-05
2	-.796E-03	0.633E-03	0.307E-04	0.143E-04	0.500E-04	0.312E-05
3	-.796E-03	0.633E-03	0.768E-04	0.143E-04	0.500E-04	0.312E-05
4	-.796E-03	0.633E-03	0.769E-04	0.143E-04	0.500E-04	0.312E-05
5	-.796E-03	0.633E-03	0.769E-04	0.143E-04	0.500E-04	0.312E-05
6	-.796E-03	0.633E-03	0.768E-04	0.143E-04	0.500E-04	0.312E-05
7	-.796E-03	0.633E-03	0.307E-04	0.143E-04	0.500E-04	0.312E-05
8	-.796E-03	0.633E-03	0.307E-04	0.143E-04	0.500E-04	0.312E-05

MECHANICAL GLOBAL PLY STRESS

PLY	SIG-X	SIG-Y	SIG-Z	SIG-YZ	SIG-XZ	SIG-XY
1	-.659E-03	0.415E-03	0.000E+00	10.0	35.0	2.18
2	-.659E-03	0.415E-03	0.000E+00	10.0	35.0	2.18
3	-.198E+04	16.5	0.000E+00	10.0	35.0	-849.
4	-.202E+04	-16.5	0.000E+00	10.0	35.0	884.

5	-202E+04	-16.5	0.000E+00	10.0	35.0	884.
6	-.198E+04	16.5	0.000E+00	10.0	35.0	-849.
7	-.659E-03	0.415E-03	0.000E+00	10.0	35.0	2.18
8	-.659E-03	0.415E-03	0.000E+00	10.0	35.0	2.18

MECHANICAL PRINCIPAL PLY STRAIN

PLY	EPS-1	EPS-2	EPS-3	EPS-23	EPS-13	EPS-12
1	-.796E-03	0.633E-03	0.307E-04	0.143E-04	0.500E-04	0.312E-05
2	-.796E-03	0.633E-03	0.307E-04	0.143E-04	0.500E-04	0.312E-05
3	-.802E-04	-.833E-04	0.768E-04	-.253E-04	0.455E-04	0.143E-02
4	-.833E-04	-.802E-04	0.769E-04	0.455E-04	0.253E-04	-.143E-02
5	-.833E-04	-.802E-04	0.769E-04	0.455E-04	0.253E-04	-.143E-02
6	-.802E-04	-.833E-04	0.768E-04	-.253E-04	0.455E-04	0.143E-02
7	-.796E-03	0.633E-03	0.307E-04	0.143E-04	0.500E-04	0.312E-05
8	-.796E-03	0.633E-03	0.307E-04	0.143E-04	0.500E-04	0.312E-05

MECHANICAL PRINCIPAL PLY STRESS

PLY	SIG-1	SIG-2	SIG-3	SIG-23	SIG-13	SIG-12
1	-.659E-03	0.415E-03	0.000E+00	10.0	35.0	2.18
2	-.659E-03	0.415E-03	0.000E+00	10.0	35.0	2.18
3	-.183E+04	-135.	0.000E+00	-17.7	31.8	1000.
4	-.190E+04	-132.	0.000E+00	31.8	17.7	*****
5	-.190E+04	-132.	0.000E+00	31.8	17.7	*****
6	-.183E+04	-135.	0.000E+00	-17.7	31.8	1000.
7	-.659E-03	0.415E-03	0.000E+00	10.0	35.0	2.18
8	-.659E-03	0.415E-03	0.000E+00	10.0	35.0	2.18

FROM OUTTHSTR.F

THERMAL GLOBAL PLY STRAIN

PLY	EPS-X	EPS-Y	EPS-Z	EPS-YZ	EPS-XZ	EPS-XY
1	-.166E-02	0.147E-02	-.707E-03	-.309E-09	-.441E-15	-.138E-21
2	-.166E-02	0.147E-02	-.707E-03	-.309E-09	-.441E-15	-.138E-21
3	-.166E-02	0.147E-02	0.897E-04	-.309E-09	-.441E-15	-.138E-21
4	-.166E-02	0.147E-02	0.897E-04	-.309E-09	-.441E-15	-.138E-21
5	-.166E-02	0.147E-02	0.897E-04	-.309E-09	-.441E-15	-.138E-21
6	-.166E-02	0.147E-02	0.897E-04	-.309E-09	-.441E-15	-.138E-21
7	-.166E-02	0.147E-02	-.707E-03	-.309E-09	-.441E-15	-.138E-21
8	-.166E-02	0.147E-02	-.707E-03	-.309E-09	-.441E-15	-.138E-21

THERMAL GLOBAL PLY STRESS

PLY	SIG-X	SIG-Y	SIG-Z	SIG-YZ	SIG-XZ	SIG-XY
1	-.533E-02	-.141E-02	0.147E-02	-.216E-03	-.309E-09	-.963E-16
2	-.533E-02	-.141E-02	0.147E-02	-.216E-03	-.309E-09	-.963E-16
3	0.129E+04	0.568E+04	0.147E-02	-.216E-03	-.309E-09	-.331E+04
4	0.129E+04	0.568E+04	0.147E-02	-.216E-03	-.309E-09	0.331E+04
5	0.129E+04	0.568E+04	0.147E-02	-.216E-03	-.309E-09	0.331E+04
6	0.129E+04	0.568E+04	0.147E-02	-.216E-03	-.309E-09	-.331E+04
7	-.533E-02	-.141E-02	0.147E-02	-.216E-03	-.309E-09	-.963E-16
8	-.533E-02	-.141E-02	0.147E-02	-.216E-03	-.309E-09	-.963E-16

THERMAL PRINCIPAL PLY STRAIN

PLY	EPS-1	EPS-2	EPS-3	EPS-23	EPS-13	EPS-12
1	-.166E-02	0.147E-02	-.707E-03	-.309E-09	-.441E-15	-.138E-21
2	-.166E-02	0.147E-02	-.707E-03	-.309E-09	-.441E-15	-.138E-21
3	-.955E-04	-.955E-04	0.897E-04	-.218E-09	-.218E-09	0.314E-02
4	-.955E-04	-.955E-04	0.897E-04	-.218E-09	0.218E-09	-.314E-02
5	-.955E-04	-.955E-04	0.897E-04	-.218E-09	0.218E-09	-.314E-02
6	-.955E-04	-.955E-04	0.897E-04	-.218E-09	-.218E-09	0.314E-02

7	-.166E-02	0.147E-02	-.707E-03	-.309E-09	-.441E-15	-.138E-21
8	-.166E-02	0.147E-02	-.707E-03	-.309E-09	-.441E-15	-.138E-21

THERMAL PRINCIPAL PLY STRESS

PLY	SIG-1	SIG-2	SIG-3	SIG-23	SIG-13	SIG-12
1	-.533E-02	-.141E-02	0.147E-02	-.216E-03	-.309E-09	-.963E-16
2	-.533E-02	-.141E-02	0.147E-02	-.216E-03	-.309E-09	-.963E-16
3	179.	0.680E+04	0.147E-02	-.153E-03	-.153E-03	0.220E+04
4	179.	0.680E+04	0.147E-02	-.153E-03	0.153E-03	-.220E+04
5	179.	0.680E+04	0.147E-02	-.153E-03	0.153E-03	-.220E+04
6	179.	0.680E+04	0.147E-02	-.153E-03	-.153E-03	0.220E+04
7	-.533E-02	-.141E-02	0.147E-02	-.216E-03	-.309E-09	-.963E-16
8	-.533E-02	-.141E-02	0.147E-02	-.216E-03	-.309E-09	-.963E-16

FROM OUTFAIL.F

ITERATION NUMBER > 3

FAILURE ASSESSMENT...

PLY > 1 MODE > X1C

PLY > 1 MODE > X2T

PLY > 2 MODE > X1C

PLY > 2 MODE > X2T

PLY > 3 MODE > S12

PLY > 4 MODE > S12

PLY > 5 MODE > S12

PLY > 6 MODE > S12

PLY > 7 MODE > X1C

PLY > 7 MODE > X2T

PLY > 8 MODE > X1C

PLY > 8 MODE > X2T

SAFTY FACTOR > 17.80445

LOCAL LAMINATE STRAIN EXCEEDS TOLERANCE...

FAILURE ANALYSIS SUMMARY:

ITERATIONS	=	4
LOAD DROPS	=	1
SF	=	95.28498
MODE	=	2
PLY	=	8.000000

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